Diyala Journal of Engineering Sciences Vol (15) No 3, 2022: 94-105

Diyala Journal of Engineering Sciences

Journal homepage: https://djes.info/index.php/djes

ISSN: 1999-8716 (Print); 2616-6909 (Online)

Analytical Approximate Solutions of Nonlinear Fractional-Order Nonhomogeneous Differential Equations

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ARTICLE INFO

Article history:
Received: July 11, 2022
Accepted: August 31, 2022

Keywords:
Adomian decomposition algorithm
MAPLE 18 software
Nonlinear fractional differential equation
Computational simulations

ABSTRACT

Computational simulation of natural phenomenon is currently attracting increasing interest in applied mathematics and computational physics. Mathematical software for simulation is limited by the availability, speed, and parallelism of high-performance computing. To improve the performance and efficiency of some numerical techniques, a step-by-step approach to mathematical software coding is needed to build robust parameter-oriented problems. Therefore, this article aims to present and apply the Adomian decomposition algorithm coded by the MAPLE 18 software package for the solutions of nonlinear fractional-order differential equations in applied physics and engineering sciences. The present technique is used without linearization or slight disturbance of nonlinear terms, which confirms the strength, accuracy, and simplicity of the algorithm. The two test problems are considered for different initial conditions and the solutions obtained show that the Adomian decomposition algorithm is fast, easy, stable in good agreement with analytical techniques and that a good computational approach to fractional-order value problems arising in applied mathematics and engineering sciences.

1. Introduction

In the past two decades, the study and analysis of fractional derivatives of ordinary differential equations, partial differential equations, systems of differential equations and integrodifferential equations have attracted more attention in the interpretation of natural phenomena of linear or nonlinear modeling science. Applications of fractional calculus are used in many fields such as electrical networks, control theory of dynamical systems, probability and statistics, electrochemistry, physical chemistry, optics, and signal processing can be successfully modeled using linear or non-linear fractional differential equations. Fractional derivatives have been applied to many physics' problems such as the frequency-dependent damping behavior of materials, the motion of a large thin plate in Newtonian fluids, and contraction and expansion functions for elastic materials and many applied mathematics sciences [1- 4].

In this article, we consider and present analytical solutions to time-fractional differential equation of the form:

$$\begin{cases} \dfrac{d^{\alpha}y}{dt^{\alpha}} + \dfrac{dy}{dt} - \gamma y^{\varphi} = g(t), \\ \qquad 0 < \alpha < 1\,, \\ \qquad \varphi > 1 \end{cases} \tag{1}$$

with the following initial conditions:

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DOI: 10.24237/djes.2022.15310

$$y(t_0) = \Omega = \begin{cases} sin\left(\frac{k}{\pi}\right) \\ cos\left(\frac{k}{\pi}\right) \quad 1 \le k \le 5 \\ log\left(\frac{k}{\pi}\right) \end{cases} \quad (2)$$

Where α fractional order of the equation, φ degree of the equation, γ inverse of the degree of the equation, $g(t)$ given function, Ω constant parameter, and k is an integer.

In recent studies, several researchers have done great works on the fractional derivative and its applications in physical sciences such as [5] obtained approximate analytical solutions of the nonlinear fractional KdV–Burgers equation, [6] presented analytical solution of time fractional two component evolutionary system of order 2 by residual power series method, [7] employed residual power series method for time fractional Schrodinger equations, construction of fractional power series solutions to fractional Boussinesq equations using residual power series method was presented by [8], [9] applied homotopy perturbation method to time fractional diffusion equation with a moving boundary condition, [10] application of Fractional variational homotopy perturbation iteration method and its application was used to solve fractional diffusion equation, [11] presented non perturbative analytical solutions of the space and time fractional Burgers equations, [12] proposed discretization schemes for fractional order differentiators and integrators, [13] proposed Trustin transform method to obtain discrete approximation of fractional order differentiator, [14] applications of fractional calculus to Newtonian mechanics analysis, [15] presented a fully discrete spectral method for the nonlinear time fractional Klein-Gordon equation, [16] discussed existence and uniqueness of solutions of initial value problems for nonlinear fractional differential equations, [17] discussed and employed a reliable algorithm of homotopy analysis method for solving nonlinear fractional differential equations, [18] used collocation method for the numerical solution of fractional differential equations, [19] proposed efficient chebyshev spectral methods for solving multi term fractional orders differential equations and [20] obtained equilibrium points, stability and numerical solutions of fractional order predator prey and rabies models, [21] applied Adomian decomposition method for solving fractional differential equations, [22] used Adomian decomposition method for a type of fractional differential equations, [23] presented a novel multistep generalized differential transform method for solving fractional order Lu chaotic and hyper-chaotic systems, an expansion iterative technique for handling fractional differential equations using fractional power series scheme proposed by [24] and Authors [25] applied homotopy perturbation Elzaki transform for the numerical solutions of time-fractional Navier-Stokes equations.

Obtaining analytical and approximate solutions to fractional differential equations is an important role in the understanding several models, except for a limited number of applied equations with difficulty in finding their analytical solutions. Therefore, the fundamental goal of this paper is to formulate three steps algorithm using Adomian decomposition method for finding approximate solutions to the nonlinear fractional-order nonhomogeneous differential equation (1) coupled with variation in initial conditions (2).

In the literature, there are several definitions of a fractional derivative of order $\alpha > 0$. In this paper, we consider the most commonly used definition of the Caputo derivative of order α as

$$\begin{cases} I^{\alpha} y(t) = \\ \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\eta)^{m-\alpha-1} y(\eta) d\eta, \end{cases} \quad (3)$$

Caputo fractional derivative allows traditional initial and boundary conditions to be included in the formulation of the problem.
From properties $D^{\alpha}$and I^{α} which leads to

$$D^{\alpha} t^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+1-\alpha)} t^{\beta-\alpha}, \quad \beta \ge \alpha \quad (4)$$

Where D^{α} is Caputo derivative operator of order α

$$I^{\alpha}t^{\beta} = \frac{\Gamma(\beta+1)}{\Gamma(\beta+\alpha+1)}t^{\beta+\alpha}, \quad (5)$$

2. Adomian decomposition method (ADM)

The Adomian decomposition method [26-29] is a powerful tool for solving linear or nonlinear equations and authors [30-31] have proved the convergences of Adomian decomposition method. We consider a nonlinear differential equation which can be decomposed into the following form

$$L(y) + R(y) + N(y) = g, \quad (6)$$

Where L is the highest order differential operator, $R(y)$ is the remainder of the linear part, $N(y)$ represents the nonlinear part and g is a given function. In general, operator L is invertible. If we take L^{-1} on both sides of equation (6) which equivalent expression can be given

$$y = -L^{-1}R(y) - L^{-1}N(y) + L^{-1}g + \Psi, \quad (7)$$

Where Ψ satisfies $L\Psi = 0$ and the initial conditions. If L is the second-order derivative, L^{-1} is the two-fold definite integral. For the Adomian decomposition method, thus, the solution y is expressed in terms of a series form:

$$y = \sum_{n=0}^{\infty} y_n \quad (8)$$

And the nonlinear term $N(y)$ is represented by the Adomian polynomials A_n as follows:

$$N(y) = \sum_{n=0}^{\infty} A_n \quad (9)$$

Equation (9) depends on $y_0, y_1, \cdots . y_n$ and can be represented as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{\infty}\lambda^k y_k\right)\right]_{\lambda=0}, \quad n = 0,1,2,\cdots \quad (10)$$

For more simplification, the first three terms of the Adomian polynomials as goes as follows

$$\begin{cases} A_0 = N(y_0), \\ A_1 = y_1 N^{(1)}(y_0), \\ A_2 = y_2 N^{(1)}(y_0) + \frac{1}{2!}{y_1}^{(2)} N^{(2)}(y_0) \\ A_3 = y_3 N^{(1)}(y_0) + y_1 y_2 N^{(2)}(y_0) + \frac{1}{3!}{y_1}^{(3)} N^{(3)}(y_0) \\ \vdots \end{cases} \quad (11)$$

Hence, equation (6) becomes

$$\sum_{n=0}^{\infty} y_n = -L^{-1}R\sum_{n=0}^{\infty} y_n - L^{-1}\sum_{n=0}^{\infty} A_n + L^{-1}g + \Psi, \quad (12)$$

And the Adomian's technique is equivalent to the following relation which defines as follows:

$$\begin{cases} y_0 = L^{-1}g + \Psi, \\ y_1 = L^{-1}R(y_0) - L^{-1}(A_0), \\ y_2 = L^{-1}R(y_1) - L^{-1}(A_1), \\ y_3 = L^{-1}R(y_2) - L^{-1}(A_2), \\ \vdots \\ y_{n-1} = L^{-1}R(y_{n-1}) - L^{-1}(A_{n-1}) \end{cases} \quad (14)$$

2.1 Five-step computational algorithm

In this section, we formulate a five-step computational algorithm using the MAPLE 18 software package by coding the Adomian decomposition method discussed in section (2) which goes thus:

Restart:
Step 1:
$withplots$:
$Digits := \mathbb{R}^+$;
$N := \mathbb{R}^+$;
$\varphi := \mathbb{R}^+$;
$g := g(t)$;
$y[0] := \Omega$;
$A[0] := \big(y[0] + int(g,t)\big)$;
$m := \alpha + \frac{1}{2}$;
Step 2:
for n from 0 to N do
$L := -diff(A[n],t)$;

$R := simplify\left(\frac{1}{\Gamma(m-\alpha)} * int((t-\eta)^{m-\alpha-1} * sub(\eta = t, L), \eta = 0 \dots t), assume = nonnegative\right);$

$H := simplify\left(-int(R + \gamma * A[n]^{\varphi}, t), assume = nonnegative\right);$

$A[n+1] := expand(H);$

end do;

Step 3:

$\boldsymbol{Sol}(\boldsymbol{algorithm}) := sum(A[k], k = 0..N+1);$

for t from 0 by 0.2 to 1 do

$y[t] := evalf(eval(\boldsymbol{Sol}(\boldsymbol{algorithm})));$

end do;

$y[t][2Dplot] := plot([case\ 1, case\ 2, case\ 3, case\ 4, case\ 5]], = 0 \dots 1, color[red, green, blue, black, yellow], axes = boxed, title =$ nonlinear fractional $IVPs);$

3. Numerical experiment

To apply this proposed algorithm, we consider two examples of the form:

Example 1. Consider nonlinear fractional-order nonhomogeneous IVP of the form

$$\begin{cases} \frac{d^{\alpha}y}{dt^{\alpha}} + \frac{dy}{dt} - \frac{1}{2}y^2 = t^2 + \frac{\Gamma(3)}{\Gamma\left(\frac{5}{2}\right)} t^{\frac{1}{2}} \\ \alpha = \frac{1}{2} \end{cases}$$

wth initial condition:

$$y(0) = \Omega = \begin{cases} cos\left(\frac{k}{\pi}\right) \\ sin\left(\frac{k}{\pi}\right) \\ log\left(\frac{k}{\pi}\right) \end{cases} \quad 1 \leq k \leq 5$$

Applying the algorithm proposed in the last section for the simulation for example 1 as stated below:

Restart:

Step 1:

$withplots:$

$Digits := 10;$

$N := 2;$

$\varphi := 2;$

$\alpha := \frac{1}{2};$

$k := [1,2,3,4,5]$

$g := t^2 + \frac{\Gamma(3)}{\Gamma\left(\frac{5}{2}\right)} t^{\frac{1}{2}};$

$y[0] := \left[cos\left(\frac{k}{\pi}\right), sin\left(\frac{k}{\pi}\right), log\left(\frac{k}{\pi}\right)\right];$

$A[0] := (y[0] + int(g, t));$

$m := \alpha + \frac{1}{2};$

Step 2:

for n from 0 to N do

$L := -diff(A[n], t);$

$R := simplify\left(\frac{1}{\Gamma(m-\alpha)} * int((t-\eta)^{m-\alpha-1} * sub(\eta = t, L), \eta = 0 \dots t), assume = nonnegative\right);$

$H := simplify\left(-int(R + \gamma * A[n]^{\varphi}, t), assume = nonnegative\right);$

$A[n+1] := expand(H);$

end do;

Step 3:

$\boldsymbol{Sol}(\boldsymbol{algorithm}) := sum(A[k], k = 0..N+1);$

for t from 0 by 0.2 to 1 do

$y[t] := evalf(eval(\boldsymbol{Sol}(\boldsymbol{algorithm})));$

end do;

$y[t][2Dplot] := plot([case\ 1, case\ 2, case\ 3, case\ 4, case\ 5]], = 0 \dots 1, color[red, green, blue, black, yellow], axes = boxed, title =$ nonlinear fractional $IVPs)$

Output: Figure 1, Figure 2, and Figure 3 and Table 1 respectively.

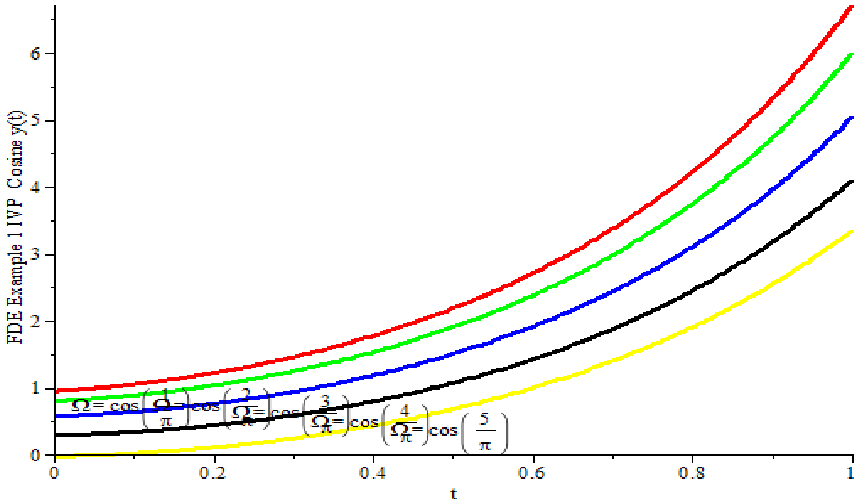

Figure 1. Depict the simulated solutions obtained when initial conditions are cosine constant parameters for $1 \leq k \leq 5$

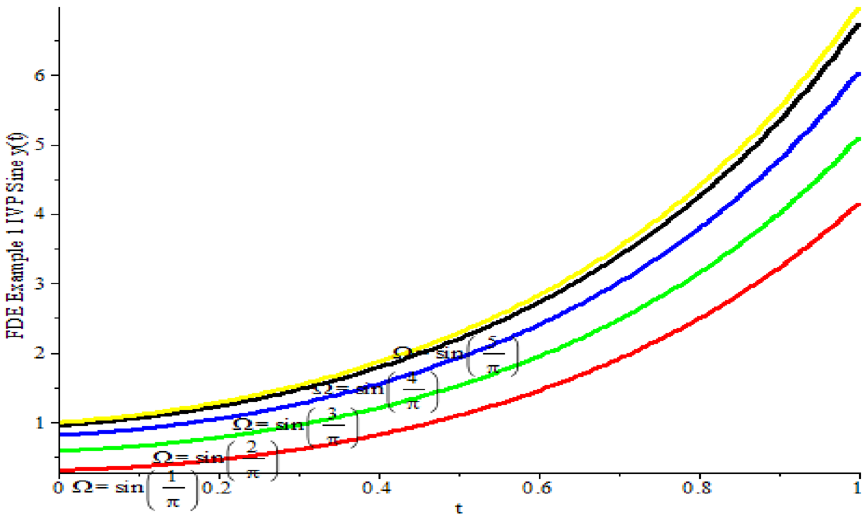

Figure 2. Depict the simulated solutions obtained when initial conditions are sine constant parameters for $1 \leq k \leq 5$

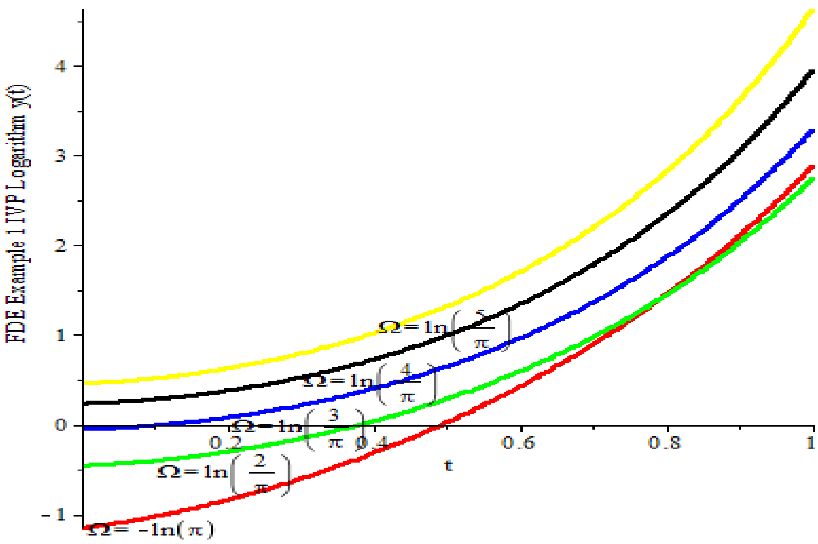

Figure 3. Depict the simulated solutions obtained when initial conditions are logarithm constant parameters for $1 \leq k \leq 5$

Table 1: Numerical solutions for Example 1 for various initial conditions

t	k	$cos\left(\frac{k}{\pi}\right)$	$sin\left(\frac{k}{\pi}\right)$	$log\left(\frac{k}{\pi}\right)$
0	1	0.9497657154	0.3129617961	-1.144729886
	2	0.8041098283	0.5944807684	-0.451582705
	3	0.5776661773	0.8162731084	-0.046117597
	4	0.2931852324	0.9560556571	0.2415644747
	5	-0.020751613	0.9997846621	0.4647080260
0.2	1	1.218004208	0.4590939223	-0.837443893
	2	1.034031472	0.7800962334	-0.297493126
	3	0.7602800719	1.0491576411	0.082803265
	4	0.4374035000	1.2260884244	0.3813181456
	5	0.6292784801	1.2826118502	0.6292784801
0.4	1	1.770959975	0.8178054980	-0.315846407
	2	1.527715571	1.2041520670	-0.041071614
	3	1.179535827	1.5474450590	0.4036464869
	4	0.7927131969	1.7818094360	0.7284753371
	5	0.4300329934	1.8580344120	1.0192607340
0.6	1	2.7048505721	1.4511210370	0.4256725341
	2	2.3717929131	1.9416919190	0.5949817489
	3	1.9096334162	2.3985110870	0.9636333938
	4	1.3420256900	2.7198860560	1.3420256900
	5	0.9930896713	2.8259391840	1.7034352750
0.8	1	4.2209763663	2.4858893580	0.4256725341
	2	3.7451685950	3.1457456910	1.4469028710
	3	3.1017955590	3.7829768090	1.8654020430
	4	2.4454266250	4.2426830140	1.3420256900
	5	1.9014446320	4.3963354010	2.8217493702
1.0	1	6.708242521	4.1525524890	1.4500867840
	2	5.987613902	5.1005945150	2.7488365860
	3	5.036496607	6.0443473150	3.2935750560
	4	3.952030994	6.7414604690	4.0954690180
	5	3.342336701	6.9774439670	4.6311944600

Example 2. Consider nonlinear fractional-order nonhomogeneous IVP of the form

$$\begin{cases} \dfrac{d^{\alpha}y}{dt^{\alpha}} - \dfrac{dy}{dt} - \dfrac{1}{2}y^2 = t^4 \\ \alpha = \dfrac{1}{2} \end{cases}$$

wth initial condition:

$$y(0) = \Omega = \begin{cases} cos\left(\frac{k}{\pi}\right) \\ sin\left(\frac{k}{\pi}\right) \\ log\left(\frac{k}{\pi}\right) \end{cases} \quad 1 \leq k \leq 5$$

Applying the algorithm proposed in the last section for the simulation for example 2 as stated below:

Restart:
Step 1:
$withplots$:
$Digits := 10;$
$N := 2;$
$\varphi := 2;$
$\alpha := \frac{1}{2};$
$k := [1,2,3,4,5]$
$g := t^4\,;$
$y[0] := \left[cos\left(\frac{k}{\pi}\right), sin\left(\frac{k}{\pi}\right), log\left(\frac{k}{\pi}\right)\right];$
$A[0] := \left(y[0] + int(g,t)\right);$
$m := \alpha + \frac{1}{2};$
Step 2:
for n from 0 to N do
$L := -diff(A[n], t);$
$R := simplify\left(\frac{1}{\Gamma(m-\alpha)} * int((t-\eta)^{m-\alpha-1} * sub(\eta = t, L), \eta = 0 \ldots t), assume = nonnegative\right);$
$H := simplify\left(\begin{matrix} int(R + \gamma * A[n]^{\varphi}, t), \\ assume = nonnegative \end{matrix}\right);$
$A[n+1] := expand(H);$
end do;

Step 3:
$\boldsymbol{Sol}(\boldsymbol{algorithm}) := sum(A[k], k = 0..N+1);$
for t from 0 by 0.2 to 1 do
$y[t] := evalf\left(eval(\boldsymbol{Sol}(\boldsymbol{algorithm}))\right);$
end do;
$y[t][2Dplot] := plot([case\ 1, case\ 2, case\ 3, case\ 4, case\ 5]], = 0 \ldots 1, color[red, green, blue, black, yellow], axes = boxed, title = \text{nonlinear fractional } IVPs)$

Output: Figure 4, Figure 5, and Figure 6 and Table 2 respectively.

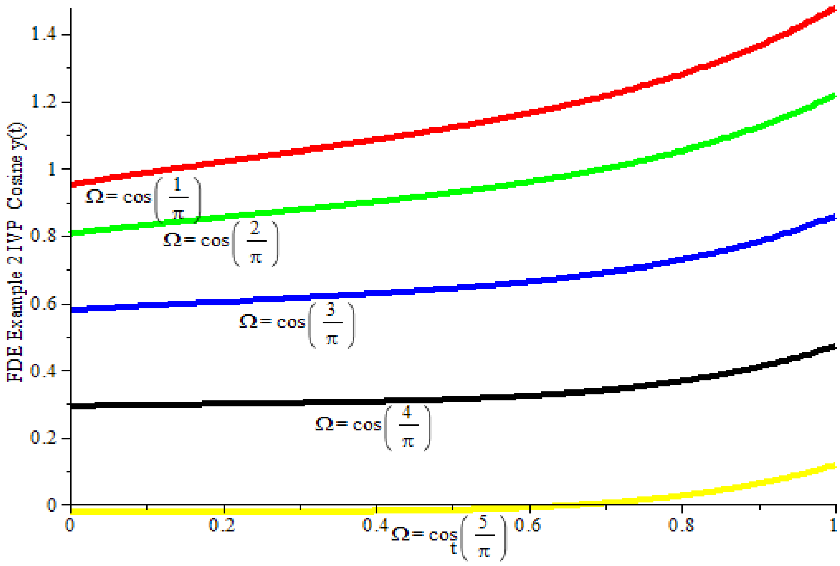

Figure 4. Depicts numerical simulation for various initial conditions for cosine functions for $1 \leq k \leq 5$

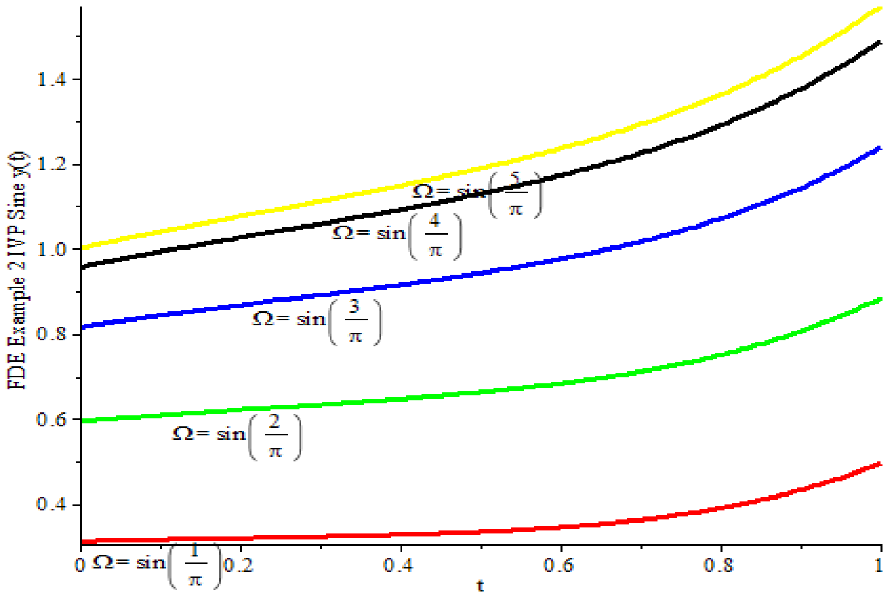

Figure 5. Depicts numerical simulation for various initial conditions for sine functions for $1 \leq k \leq 5$

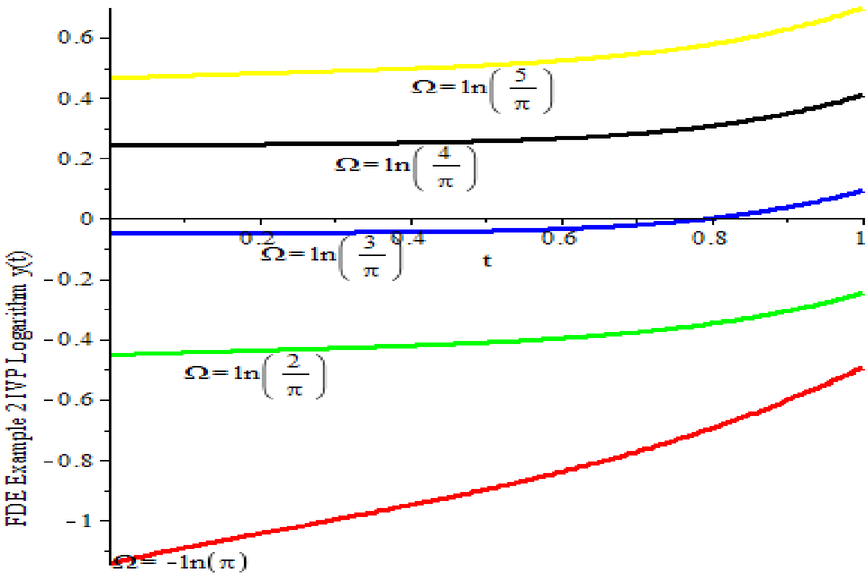

Figure 6. Depict the simulated solutions obtained when initial conditions are logarithm constant parameters for $1 \leq k \leq 5$

Table 2: Numerical solutions for Example 2 for various initial conditions

t	k	$cos\left(\frac{k}{\pi}\right)$	$sin\left(\frac{k}{\pi}\right)$	$log\left(\frac{k}{\pi}\right)$
0	1	0.9497657154	0.31296179615	- 1.1447298869
	2	0.8041098283	0.59448076840	- 0.4515827056
	3	0.5776661773	0.81627310840	- 0.0461175974
	4	0.2931852324	0.95605565711	0.24156447471
	5	-0.020751613	0.99978466211	0.46470802601
0.2	1	1.0189319730	0.32049784586	- 1.0441298429
	2	0.8536567183	0.62155678890	- 0.4359463627
	3	0.6032333568	0.86733233521	- 0.0459014434

	4	0.2998052740	1.02614345610	0.24607548191
	5	-0.020664933	1.07644973312	0.48126581121
0.4	1	1.0839557920	0.32881615846	- 0.9496242929
	2	0.9004002796	0.64762677181	- 0.4203910285
	3	0.6279252496	0.91547738831	- 0.0442031909
	4	0.3072961480	1.09203117410	0.25166245380
	5	-0.019080079	1.14851692610	0.49775502661
0.6	1	1.1627421641	0.34611453109	- 0.8398578034
	2	0.9589839452	0.68381021342	- 0.3963299888
	3	0.6626322846	0.97559689821	- 0.0340883091
	4	0.3237183252	1.17178128610	0.26606616221
	5	-0.009056119	1.23518379210	0.52371422481
0.8	1	1.2795957050	0.39062683550	- 0.6955402624
	2	1.0517422660	0.75028808391	- 0.3477705053
	3	0.7273924173	1.07017382220	0.00116561785
	4	0.3671990212	1.28979336010	0.30715025930
	5	0.0262101683	1.36153383410	0.57831639291
1.0	1	1.4757611510	0.49618569948	- 0.4944171713
	2	1.2177815410	0.88361444090	- 0.2474740536
	3	0.8585783483	1.23848098891	0.09233499780
	4	0.4713977064	1.48741157710	0.40812812491
	5	0.1177114856	1.56961838410	0.69678728951

4. Results and discussion

To demonstrate the accuracy and efficiency of the analytic-numerical technique presented, two examples are considered and from the computational simulation results and graphs obtained, we observed as follows:

i. The proposed algorithm demonstrated fast convergence with no linearization of the nonlinear fractional nonhomogeneous differential equation (1) considered.
ii. Example 1. The highest solutions were obtained when the initial condition was $cos\left(\frac{1}{\pi}\right)$ (red) and the least solutions are recorded $sin\left(\frac{1}{\pi}\right)$ (red) and $log\left(\frac{1}{\pi}\right)$ (red) (Figure 1, Figure 2 and Figure 3 respectively).
iii. Example 2. The lowest solutions were obtained when the initial condition was $cos\left(\frac{5}{\pi}\right)$ (yellow) and the highest solutions are recorded $sin\left(\frac{5}{\pi}\right)$ (yellow) and $log\left(\frac{5}{\pi}\right)$ (yellow) (Figure 4, Figure 5, and Figure 6 respectively).
iv. Every simulation and computation work are done using the MAPLE 18 software package.

5. Conclusions

In this paper, Adomian decomposition method was successfully coded with the MAPLE 18 software package and applied to solve nonlinear fractional differential equation arises in applied physics and engineering sciences. The fractional derivatives described in the Caputo sense which is obtained by Riemann-Liouville fractional integral operator are considered to obtain the analytic-numeric solutions. Three test case problems are considered to demonstrate the efficiency of the formulated algorithm and the best results are obtained in the third step $y(t) = \sum_{t=0}^{3} y_t$. It is concluded that the adomian decomposition algorithm is a powerful, efficient, and reliable tool for the analytical solutions of nonlinear fractional ordinary differential equations arising in applied mathematics.

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APPENDIX

Step 1:

$restart:$
$with(linalg):$
$with(plots):$
$\mathbf{Digits := 10;}$

$$g := \left(t^2 + \frac{(\Gamma(3))}{\left(\Gamma\left(\frac{5}{2}\right)\right)} \cdot t^{\frac{1}{2}} \right);$$

$N := 2;$

$\alpha := \frac{1}{2};$

$y[0] := \Omega;$
$A[0] := y[0] + int(g, t);$

$\alpha := \frac{1}{2};$

$m := \alpha + \frac{1}{2};$

$$Digits := 10$$

$$g := t^2 + \frac{8}{3}\,\frac{\sqrt{t}}{\sqrt{\pi}}$$

$$N := 2$$

$$\alpha := \frac{1}{2}$$

$$y_0 := \Omega$$

$$A_0 := \Omega + \frac{1}{3}\,t^3 + \frac{16}{9}\,\frac{t^{3/2}}{\sqrt{\pi}}$$

$$\alpha := \frac{1}{2}$$

Step 2:

$\mathbf{for}\ n\ \mathbf{from}\ 0\ \mathbf{to}\ N\ \mathbf{do}$
$L := -diff(A[n], t):$
$R := simplify$
$\left(\frac{1}{\mathrm{GAMMA}(m-\alpha)} \cdot int\left((t-\eta)^{m-\alpha-1} \cdot subs(t=\eta, L), \eta = 0..t \right), 'assume = nonnegative' \right):$

$H := simplify\left(-int\left(R - \frac{1}{2} \cdot (A[n])^2, t\$m \right), 'assume = nonnegative' \right):$
$A[n+1] := expand(H):$
$\mathbf{end\ do:}$
$Y := sum(A[k], k = 0..N+1):$

Step 3:

$T := eval\left(Y, \left[\Omega = \cos\left(\frac{1}{\pi} \right) \right] \right):$ for x from 0 by 0.2 to 1 do $ADM := evalf(eval(T, t=x))$ end do;

0.9497657154

1.218004208

1.770959975

2.704850572

4.220976366

6.708242521

$T := eval\left(Y, \left[\Omega = \sin\left(\frac{1}{\pi} \right) \right] \right):$ for x from 0 by 0.2 to 1 do $ADM := evalf(eval(T, t=x))$ end do;

0.3129617961

0.4590939223

0.8178054980

1.451121037

2.485889358

4.152552489

$T := eval\left(Y, \left[\Omega = \log\left(\frac{1}{\pi} \right) \right] \right):$ for x from 0 by 0.2 to 1 do $ADM := evalf(eval(T, t=x))$ end do;

$$-1.144729886$$
$$-0.8374438933$$
$$-0.3158464070$$
$$0.4256725341$$
$$1.450086784$$
$$2.884962447$$

$$Case[1] := eval\left(Y, \left[\Omega = \cos\left(\frac{1}{\pi}\right)\right]\right):$$
$$Case[2] := eval\left(Y, \left[\Omega = \cos\left(\frac{2}{\pi}\right)\right]\right):$$
$$Case[3] := eval\left(Y, \left[\Omega = \cos\left(\frac{3}{\pi}\right)\right]\right):$$
$$Case[4] := eval\left(Y, \left[\Omega = \cos\left(\frac{4}{\pi}\right)\right]\right):$$
$$Case[5] := eval\left(Y, \left[\Omega = \cos\left(\frac{5}{\pi}\right)\right]\right):$$

$$Case[1] := eval\left(Y, \left[\Omega = \sin\left(\frac{1}{\pi}\right)\right]\right):$$
$$Case[2] := eval\left(Y, \left[\Omega = \sin\left(\frac{2}{\pi}\right)\right]\right):$$
$$Case[3] := eval\left(Y, \left[\Omega = \sin\left(\frac{3}{\pi}\right)\right]\right):$$
$$Case[4] := eval\left(Y, \left[\Omega = \sin\left(\frac{4}{\pi}\right)\right]\right):$$
$$Case[5] := eval\left(Y, \left[\Omega = \sin\left(\frac{5}{\pi}\right)\right]\right):$$

$$Case[1] := eval\left(Y, \left[\Omega = \log\left(\frac{1}{\pi}\right)\right]\right):$$
$$Case[2] := eval\left(Y, \left[\Omega = \log\left(\frac{2}{\pi}\right)\right]\right):$$
$$Case[3] := eval\left(Y, \left[\Omega = \log\left(\frac{3}{\pi}\right)\right]\right):$$
$$Case[4] := eval\left(Y, \left[\Omega = \log\left(\frac{4}{\pi}\right)\right]\right):$$
$$Case[5] := eval\left(Y, \left[\Omega = \log\left(\frac{5}{\pi}\right)\right]\right):$$

plot([*Case*[1], *Case*[2], *Case*[3], *Case*[4], *Case*[5]], ***t* = 0 ..1**, ***color*** = [***red, green, blue, blacl***
yellow], *labels* = ["t", " FDEProblem 1 IVP y(t)"], *labeldirections* = [*HORIZONTAL*,
VERTICAL], ***axes = BOXED, title*** = "Nonlinear Fractional order IVP Example 1 sine ")

plot([*Case*[1], *Case*[2], *Case*[3], *Case*[4], *Case*[5]], ***t* = 0 ..1**, ***color*** = [***red, green, blue, bl***
yellow], *labels* = ["t", " FDEProblem 1 IVP y(t)"], *labeldirections* = [*HORIZONTAL*,
VERTICAL], ***axes = BOXED, title*** = "Nonlinear Fractional order IVP Example 1 sine ")

plot([*Case*[1], *Case*[2], *Case*[3], *Case*[4], *Case*[5]], ***t* = 0 ..1**, ***color*** = [***red, green, blue, black,***
yellow], *labels* = ["t", " FDEProblem 1 IVP y(t)"], *labeldirections* = [*HORIZONTAL*,
VERTICAL], ***axes = BOXED, title*** = "Nonlinear Fractional order IVP Example 1 sine ")