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Behavior of Deep Beams under Three-Point Loading – A Nonlinear Finite Element Investigation

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ABSTRACT

Abstract. Deep beams transfer loads primarily through short, steep compression struts rather than classic flexural action; their response is highly sensitive to how forces enter the member and to the support layout. In tanks, transfer girders, and podium edges, midspan three-point loading is common, and in practice those loads are rarely equal. This study interrogates that everyday imbalance. Even when applied at midspan, unequal loading is a major driver of deep-beam response and capacity. By contrast, with a central, equal load, the compression-strut paths that carry force from the supports to the loading node(s) are typically symmetric. However, in the present study, the load inequality causes these struts to be asymmetrical. As a result, the strut carrying the larger load failed before the one carrying the smaller load. Therefore, the deep beam fails early. Twenty-one deep beam specimens were analyzed using SAP 2000 software, which is based on the well-known finite element method. Three patterns of load distribution between three concentrated load points were adopted: 33%-33%-33%, 50%-25%-25%, 25%-50%-25%, 67%-16.5%-16.5%, 16.5%-67%-16.5%, 75%-12.5%-12.5% and 12.5%-75%-12.5%. These load cases were studied using different concrete's compressive strength values of 20, 30, and 40 MPa. Based on these results, load capacity remained essentially unchanged, while midspan deflection and shear stresses decreased by 4.0–4.6% and 3.8–17%, respectively; in contrast, the maximum positive moments increased by 0.55–7%.

1. Introduction

In building and bridge construction, reinforced concrete (RC) deep beams are commonly used to transmit vertical loads to the foundation when there is a discontinuity in the load path. Because of its deep geometry, RC deep beams behave quite differently from typical RC beams. When the span to depth ratio is less than 4 and/or shear span (a) equal or less than total height (h) (ACI 318–14), RC beams are often categorized as deep beams [1]. In addition to supporting and loading boundary conditions, a variety of parameters, including the strengths of the concrete and steel reinforcement, can affect how RC deep beams behave [2-4]. Deep beams represent one of the D-regions at which significant disturbance in stresses occur. D-regions are zones that have undergone geometrical modifications or concentrated forces. The premise that the "plane section remains plane" is broken here. In contrast, B-regions act like typical beams. Their linear strain distribution, which adheres to Bernoulli's principle, makes beam theory

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analysis easier [5, 6]., Among RC deep beam failures, shear failure is one of the most significant. Shear failure in deep members made of reinforced concrete is brittle and happens quickly and unexpectedly. A number of studies have looked at the shear bearing capacity of reinforced concrete members both theoretically and experimentally [7-10]. It has been discovered that a number of processes, such as the dowel action of longitudinal reinforcing bars spanning the crack, stirrups crossing the shear crack, aggregate interlock across the crack face, and shear transfer in the compression zone, may help to provide shear resistance in reinforced concrete beams [11]. Boundary conditions have a significant impact on the behavior of RC deep beams. For example, at both deep and shallow beams, web reinforcement plays an important role, but its importance is even greater at deep beams. The reason is that shallow beams suffer from the combined effect of moments and shear, while in deep beams the web reinforcement strengthens the struts that transfer the load directly from the loading to the supporting points [12]. Continuous RC deep beams and simply supported deep beams have distinct failure scenarios. There is a dearth of study on RC deep beams with continuous boundary conditions, despite the fact that several numerical models have been created to evaluate the load capacity of RC deep beams under simply supported conditions. Two main models have been developed to evaluate the load capacity of continuous reinforced concrete deep beams: the ultimate strength method [13, 14] and the Strut-and-Tie Model (STM) [15]. The basic distinctions between these two models are found in how they view the distribution of compressive stresses. While the STM believes that compressive loads are localized along the concrete struts, the ultimate strength method focuses on figuring out how much load a beam can support before failing. The behavior of continuous RC deep beams may often be better predicted by the STM due to the very irregular stress distribution found in these types of deep beams. Guidance on how much load is carried by truss vs. arch action is limited. To avoid overly flat struts, ACI 318-19 and AASHTO LRFD [16] require a minimum 25° angle between struts and ties. When $a/d > 2.14$, the truss idealization must include vertical ties. According to the 1999 FIP recommendations [17], the shear span-depth ratio, which is given by $(2a/d - 1)/3$, determines the percentage of force that the truss action resists. As a result, in continuous deep beams, the truss action transfers all shear for $a/d > 2$, whereas the arch action transfers all shear for $a/d < 1/2$. According to Foster and Gilbert [18], the truss-action share of the load can be estimated as $(\sqrt{3}a/d - 1)/2$. In essence, all shear is delivered by arch action for $a/d < 0.58$ and all force is transferred by truss action for $a/d > 1.73$. According to Brown and Bayrak's strain energy analysis [19], the best way to resist loads for beams with a/d less than 2 is to place a single straight strut between the load point and the support. An excellent resource for studying and designing reinforced concrete members with D-regions is the strut and tie model [20, 21, 22, 23]. Many studies have examined how beam width, overall depth, and concrete compressive strength ($f'c$) influence the strength and response of reinforced-concrete deep beams. Because a deeper beam expands the size of the struts, the researchers concluded that the load capacity of the beam increases as it widens. The load capacity increases as the depth increases and the shear span decreases. By enhancing the concrete's resistance to compression, deep beams may support heavier loads, which also requires additional strut strengthening [24]. Decoupling the truss action from the arch action in deep beams has not been approached analytically very often. By assuming that the cross-sectional area and the modulus of elasticity are constant and equal for every component, Matamoros and Wong [25] employed the stiffness approach to solve the statically indeterminate model, which combines these two processes. Kim and Jeon [26] proposed a model to quantify the arch-action share of shear capacity in shear-critical RC beams, derived from a sectional compatibility requirement for shear deformation. According to a criterion for optimizing the combined model's strength capacity, He et al. [27] calculated the contribution of each process. In order to disentangle the arch and beam activities in reinforced concrete beams, Nakamura et al. [28] devised a numerical Rigid-Body-Spring-Method (RBSM).

The unequal applied load, even if it is central, is either due to architectural intent or an implementation error. This unequal load leads to asymmetry in the struts carrying the load from the loading to the supporting points. That leads to the failure of the weakest strut or the strut with the greatest load. Because the STM is within the lower bound theory, the first failure that occurs is the ruling, so this topic under study is of great importance. The deep-beam design followed ACI 318-19, whereas the finite-element model was developed in SAP2000.

2.1 Finite Element Modeling with SAP2000 Software

The simply supported RC deep beam was designed in accordance with ACI 318-19, see Figure 1. A single beam had 118 of finite elements. This number was chosen after the best number was examined in order to guarantee the required precision while requiring the least amount of time and effort. Reinforcing steel bars were intended to act in both compression and tension as an elastic, fully plastic material. In the current research, twenty-one simply supported reinforced concrete deep beam specimens subjected to three concentrated loads were analyzed, see Figure 2. The total load (the summation of the three loads =100%) was distributed into three loads of varying percentages in succession in order to cover the most influential cases of unequal loading, see Table 1.

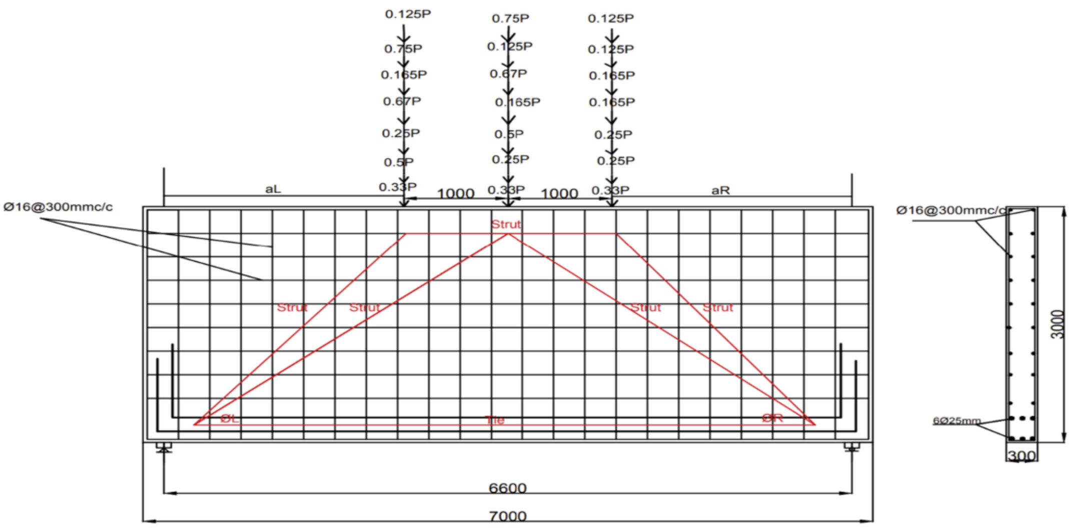


Figure 1. Deep-beam geometry, reinforcement layout, and loading scheme (units: mm)

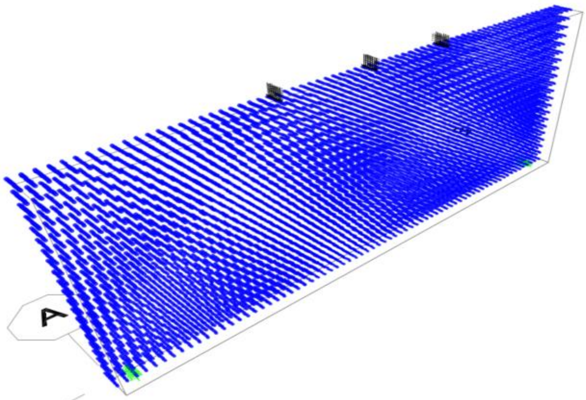
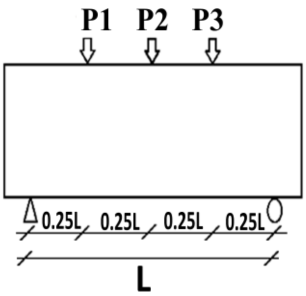


Figure 2. Deep-beam finite-element model (SAP2000)

No	Group					Sketch
1	A					
2						
3						
4						
5						
6			75%	12.5%	12.5%	
7			12.5%	75%	12.5%	
8	B	30	33.3%	33.3%	33.3%	
9			50%	25%	25%	
10			25%	50%	25%	
11			67%	16.5%	16.5%	
12			16.5%	67%	16.5%	
13			75%	12.5%	12.5%	
14			12.5%	75%	12.5%	
15	C	40	33.3%	33.3%	33.3%	
16			50%	25%	25%	
17			25%	50%	25%	
18			67%	16.5%	16.5%	
19			16.5%	67%	16.5%	
20			75%	12.5%	12.5%	
21			12.5%	75%	12.5%	

2.2 Verification of SAP 2000 analysis results

The three simply supported beams below, which were obtained from the reference [24], were reanalysed here in order to validate the SAP 2000 analysis results, Figures 3 and 4. The results indicate that the model is accurate; the largest discrepancy reported in Table 2 is under 10%. Figure 5 shows the load-deflection response for laboratory and numerical results. That results encouraged the authors to move forward with the current research.

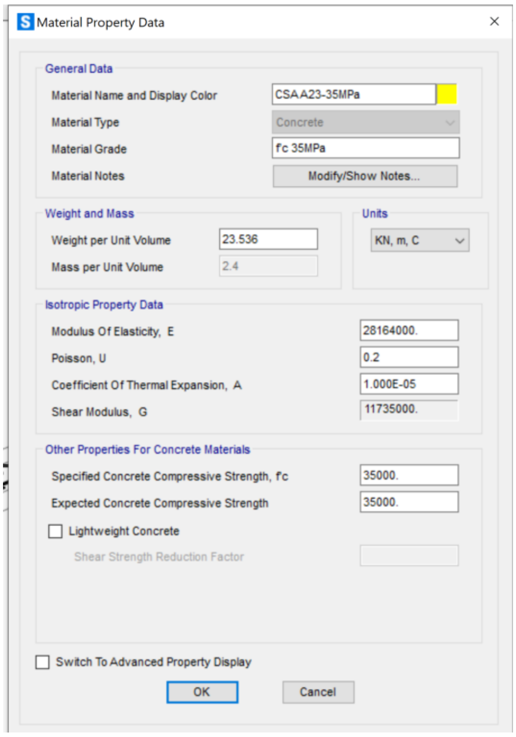


Figure.3-a: Material Properties

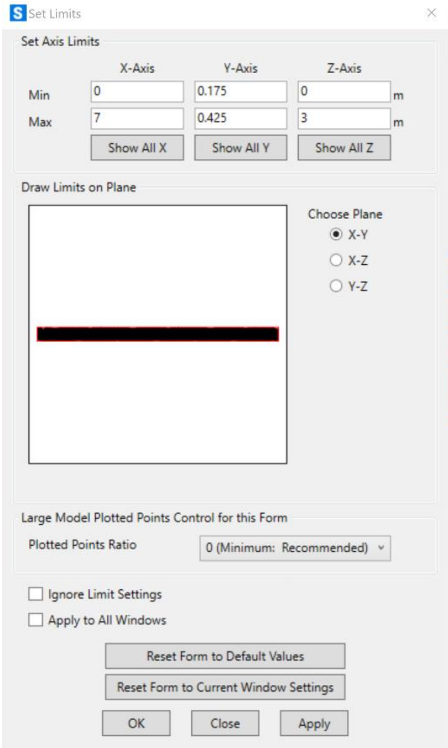
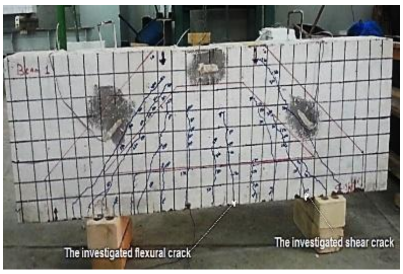


Figure 3-b:Set Axis limits

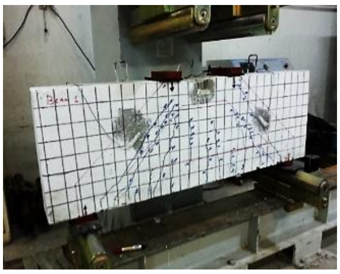
Figure 3: SAP 2000 Analysis Process

Table 2. Finite-Element Model Validation

No.	Load Type	Experimental [24]		Numerical		Difference	
		Load Capacity (kN)	Midspan Deflection (mm)	Load Capacity (kN)	Midspan Deflection (mm)	Load Capacity	Midspan Deflection
1	Single load	355	5.73	384	6.1	8%	6%
2	2 point load	562	8.62	586	9.26	4.3%	7.4%
3	Uniform load	547.8	7.57	582	8.23	6%	8.7%



1-Point load



2-Point load

Uniform load

Figure 4. Experimental validation of SAP2000 finite-element results, [24]

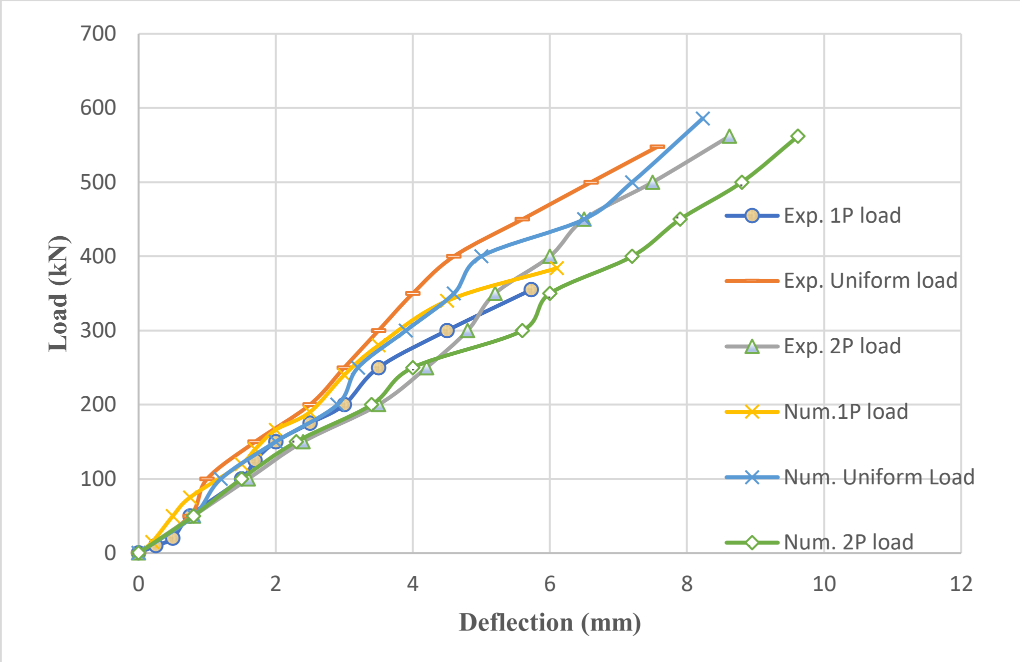


Figure 5. Load–deflection response: laboratory vs SAP2000

2.3 Simply Supported Deep Beam Study Cases

The beam is 7000 mm in length, 6600 mm in clear span, 300 mm in width, and 3000 mm in height. Each beam satisfied the minimum reinforcement ratios stipulated in ACI 318-19 [15]. The main reinforcement consisted of 6ϕ25 mm steel bars, whereas the shear reinforcement was ϕ16mm@300mm center/center. As seen in Table 1, these twenty-one deep beams are distributed among three equal groups: A, B, and C. There are three groups with different compressive strength values: 20 MPa for the first group, 30 MPa for the second, and 40 MPa for the third.

2.4 Material Properties

Deep beams were modeled with separate material definitions for concrete and reinforcing steel. For each beam, the appropriate element types were assigned to represent these materials. Table 3 lists the material parameters—Poisson's ratio (*v*), concrete compressive strength (*f'c*), steel modulus of elasticity (*Es*), concrete modulus of elasticity (*Ec*), and the yield stress (*fy*) of the flexural and web reinforcement—and Figure 6 provides the related illustration.

Table 3. Material properties of the specimens

Concrete			Reinforcing Steel		
Ec *** (MPa)**	***f'c*** **(MPa)**	**Poisson`s ratio(*v*)**	***fy*** **(MPa)**	***fys*** **(MPa)**	***Es*** **(MPa)**
21019	20	0.21	440	420	200000
25743	30				
29725	40				

$*Ec = 4700\sqrt{f_c'}$

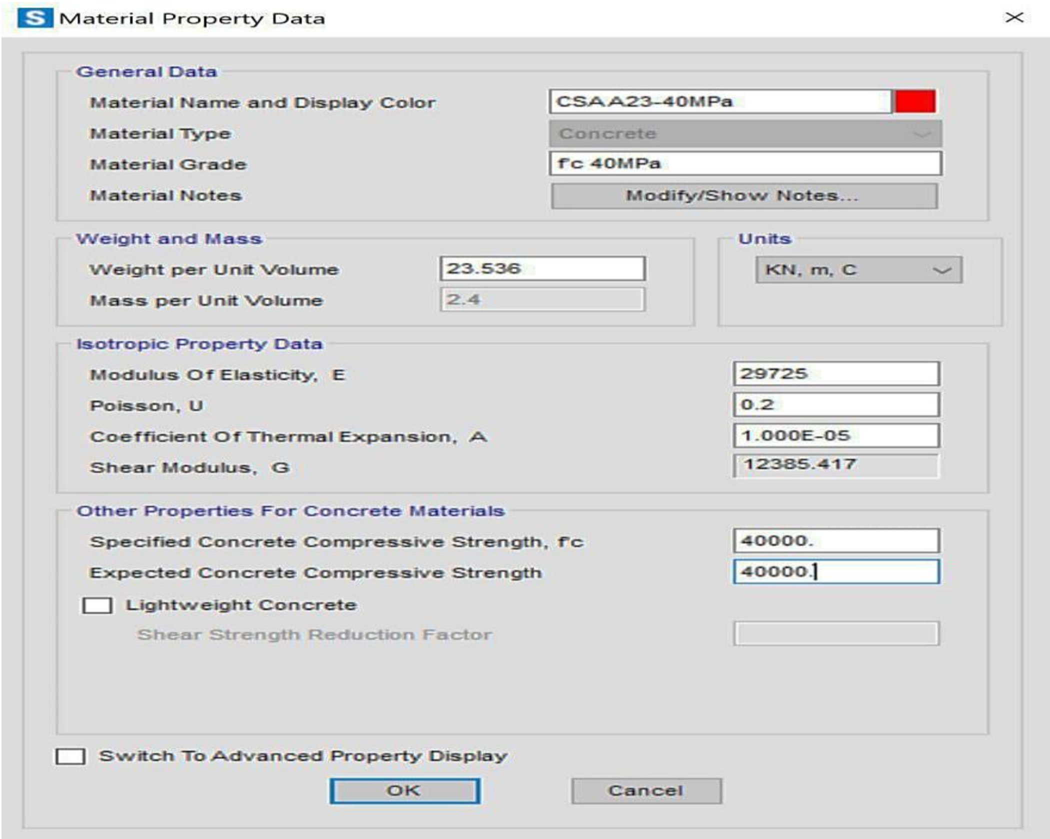


Figure 6. Mechanical properties of constituent materials

3. The effect of unequal three-point loading values

SAP2000 software was used to calculate the maximum load capacity, maximum positive moments, maximum shear stresses, and maximum midspan deflection of each beam. The detailed analysis of deep beams for the present parametric study is displayed in Table 4. From this detailed analysis, it is seen that when the load is equal (33%, 33%, 33%), the highest deflection and the highest positive moments are in the center of the beam, while the highest shear remains at one of the supports (without any difference). However, if the highest load is extreme, such as (75%, 12.5%, 12.5%), the load decreases, accompanied by a decrease in deflection (due to failure of the strut closest to the highest load). The maximum positive moments increase proportionally under the highest load position, while the maximum shear decreases and its location is at the support closest to the highest load.

Since deep beam capacity is largely governed by the compression strut—and that strut's strength scales with the concrete compressive strength (*f'c*)— the influence of load distribution by varying *f'c* was examined as follows:

3.1 Unequal-load influence for f'c = 20 MPa (Group A)

When the load is not equal among the three loads imposed on the 20 MPa concrete deep beam, i.e. the highest load is 50%, 67% and 75% being away from the center of the beam to its edge, this casts a shadow on the results of the numerical analysis.

The following were the outcomes:

1-Load capacity decreased by 4.3%, 6%, and 5% as shown in Figure 7.

2-Maximum midspan deflection decreased by 4%, 4% and 1.3% as shown in Figure 8.

3-The maximum positive moment decreased by 5.2%, 4.5% and 5% as shown in Figure 9.

4-Maximum shear stresses decrease by 9.2%, 13%, and 14.5 % as shown in Figure 10.

3.2 Unequal-load influence for f'c = 30 MPa (Group B)

The findings of the numerical analysis are affected when the three-point loading placed on the 30 MPa concrete deep beam are not equal, that is, when the highest load is 75%, 67%, and 50% traveling between the center of the beam and its edge. The results were as follows:

1. Load capacity decreased by 6%, 11% and 9% as shown in Figure 11.
2. Maximum midspan deflection decreased by 6%, 6.4% and 4.4% as shown in Figure 12.
3. The maximum positive moment decreased by 7%, 6%, and 3.1% as shown in Figure 13.
4. Maximum shear stresses decrease by 10%, 17%, and 17% as shown in Figure 14.

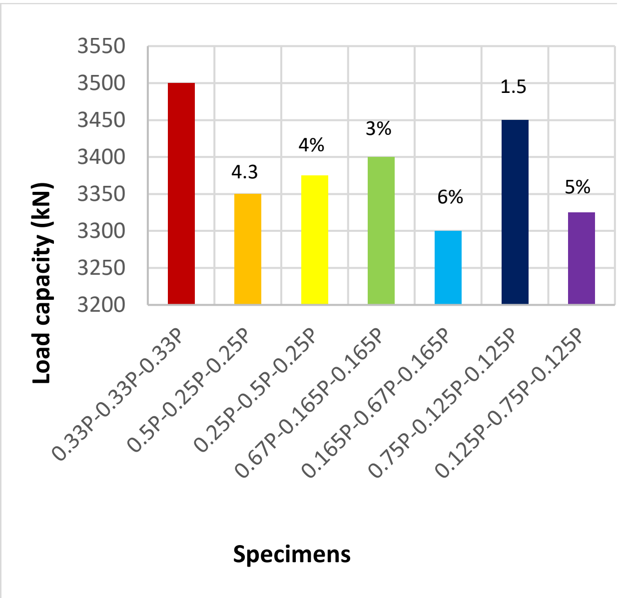


Figure 7. Load capacity vs. unequal load for group A

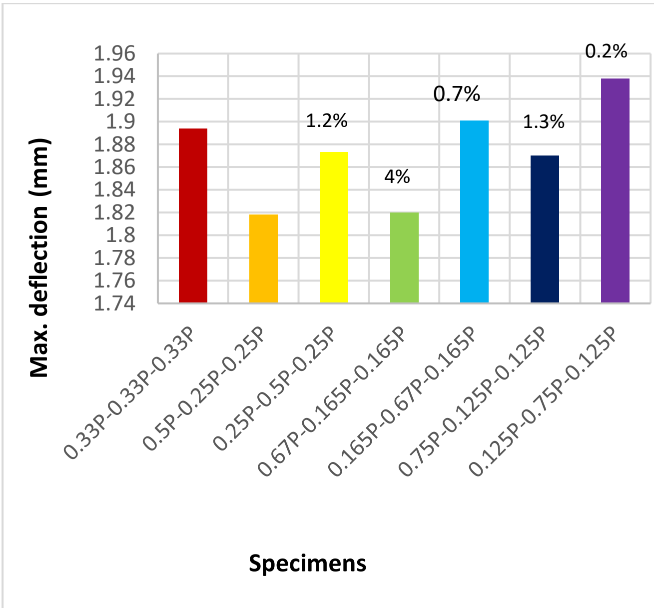
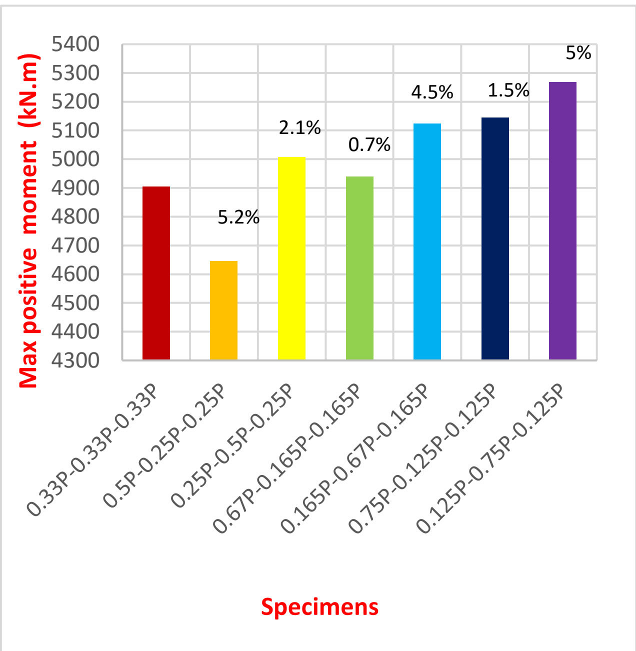


Figure 8. Max. deflection vs. unequal load for group A



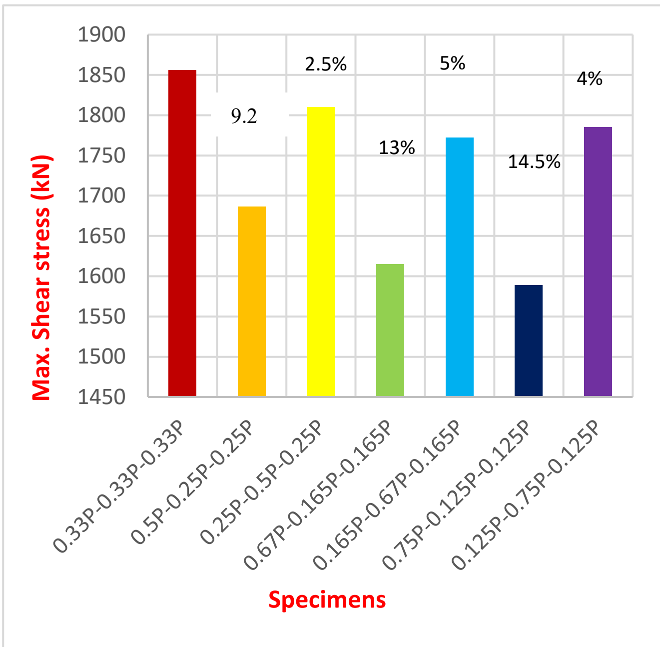
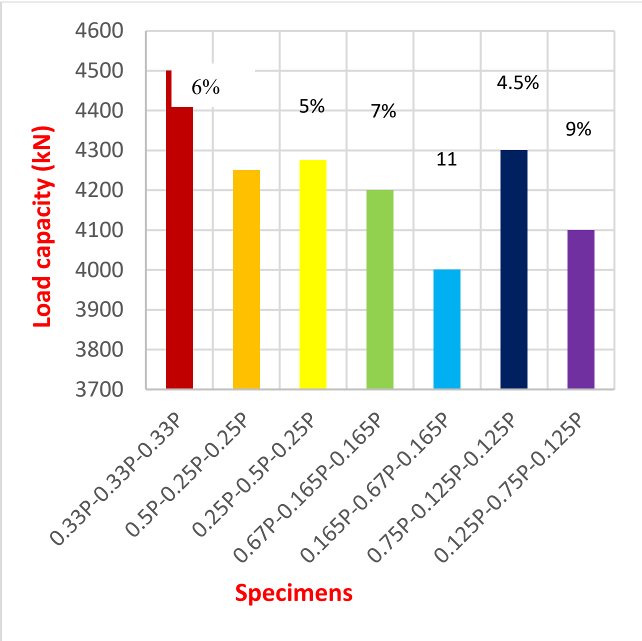


Figure 9. Group A: peak bending moment under unbalanced loading

Figure 10. Group A: peak shear stress under unbalanced loading



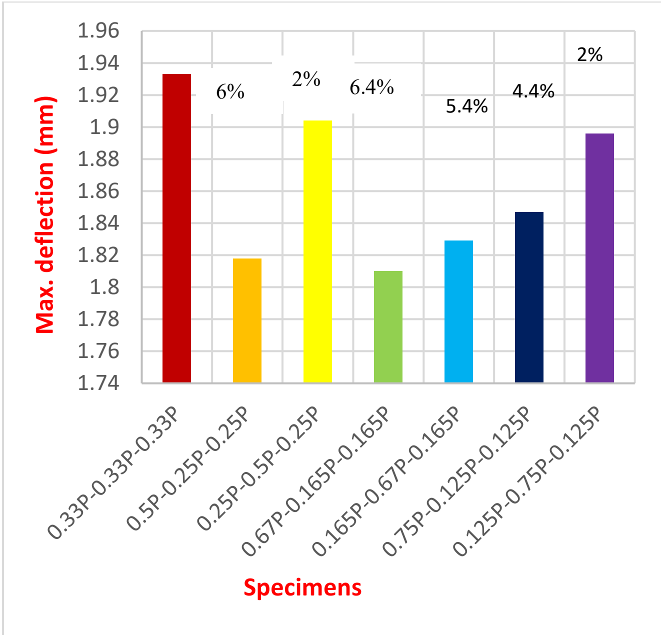
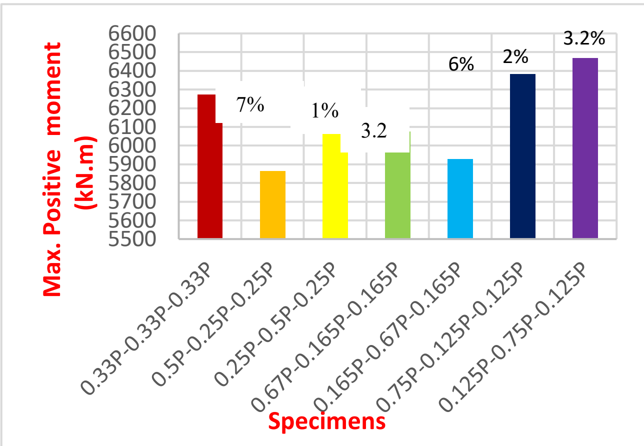


Figure 11. Group B: load-carrying capacity under unbalanced loading

Figure 12. Group B: peak deflection under unbalanced loading



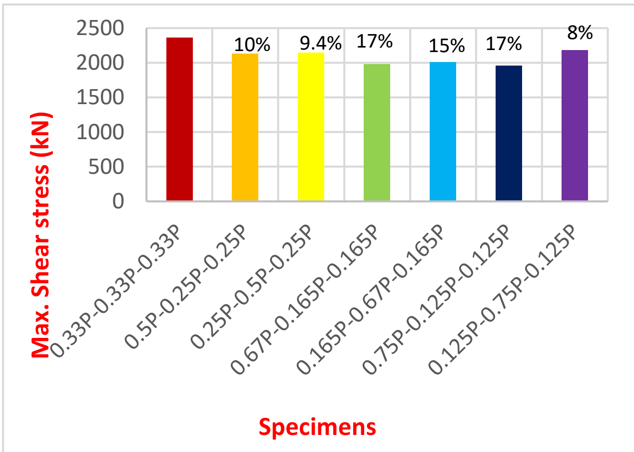


Figure 13. Group B: peak bending moment under unbalanced loading

Figure 14. Group B: peak shear stress under unbalanced loading

3.3 Unequal-load influence for f'c = 40 MPa (Group C)

The numerical analysis's findings are clouded when the three loads placed on the 40 MPa concrete deep beam are not equal, that is, when the highest load is 75%, 67%, or 50% traveling between the beam's center and edge. These were the results that were obtained:

1-Load capacity decreased by 3.2%, 4.2% and 6.3% as shown in Figure 15.

2-Maximum Midspan deflection decreased by 4%, 4.2 and 0.2% as shown in Figure16.

3-Maximum positive moment decreased by 5.4%, 5.1% and 5% as shown in Figure 17.

4-Maximum shear stresses decrease by 3%, 8.1%, and 8.1 % as shown in Figure 18.

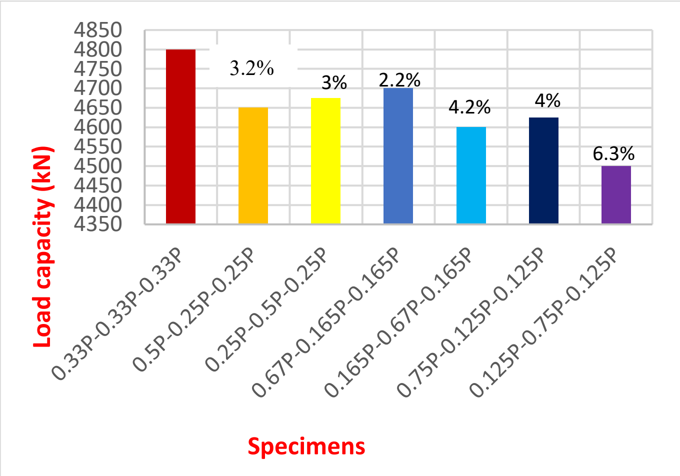


Figure15. Group C: load-carrying capacity under unbalanced loading

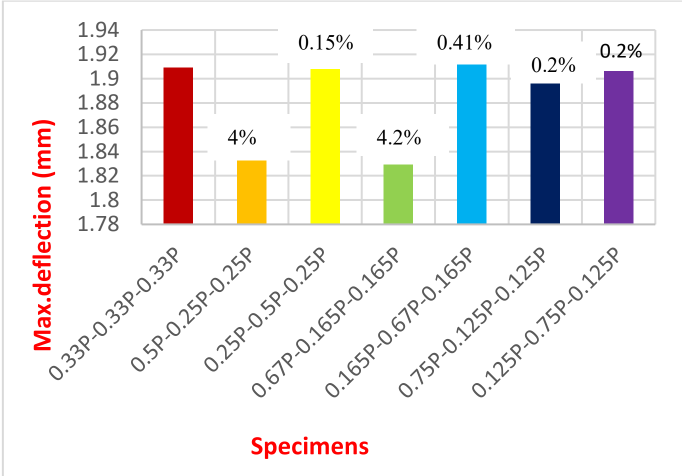


Figure 16. Group C: peak deflection under unbalanced loading

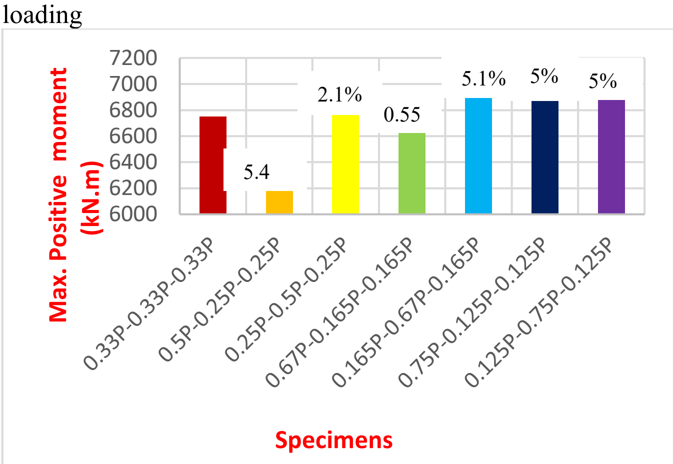


Figure 17. Group C: peak bending moment under unbalanced loading

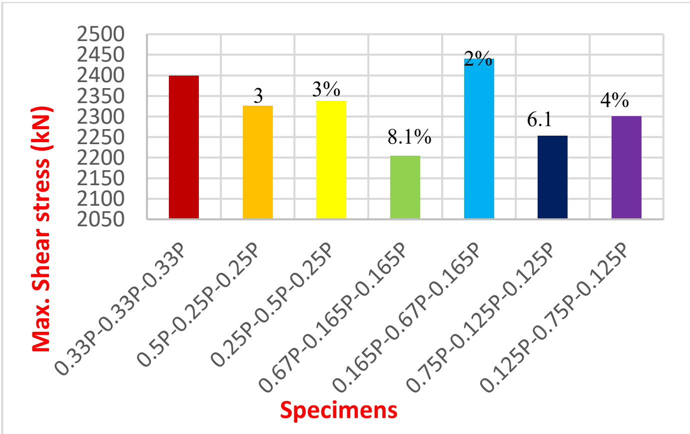


Figure 18. Group C: peak shear stress under unbalanced loading

3.4 The effect of concrete strength on load inequality:

Because the forces in deep beams are transmitted directly from the loading to the supporting points through the struts, and because they are compressive structural members similar to columns, the resistance of concrete to compression has an important effect. Therefore, the concrete's resistance to compression was changed here, which clearly improved the load capacity and deflection together, and also increased the maximum positive moment and shear values. More specifically, when the concrete compressive strength increased from 20 to 40 MPa, the load capacity, maximum positive moments and maximum shear stresses increased by about 29-37%. As for the deflection, its values did not increase significantly, but it improved compared to the increase in load that accompanies it, as illustrated in Figures 19-22.

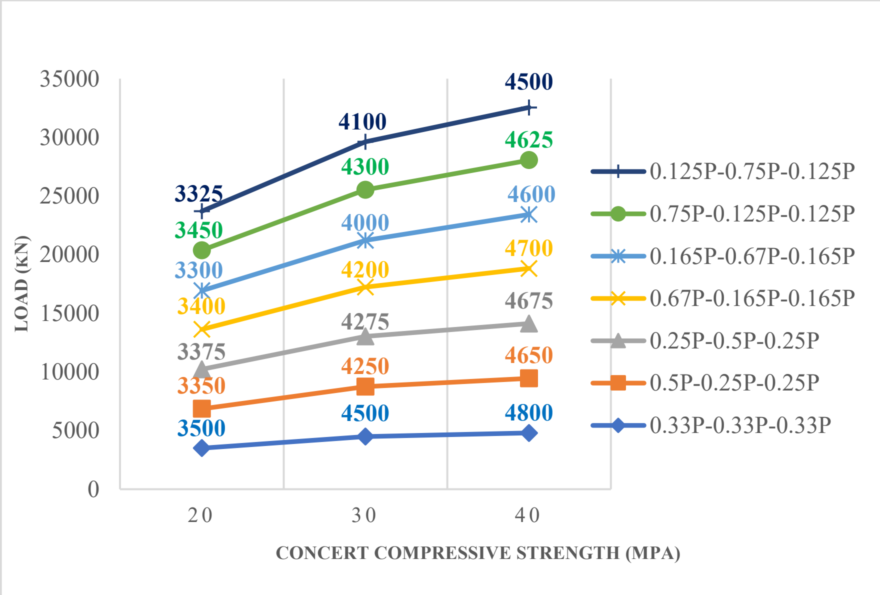


Figure19. Capacity vs concrete compressive strength (all groups)

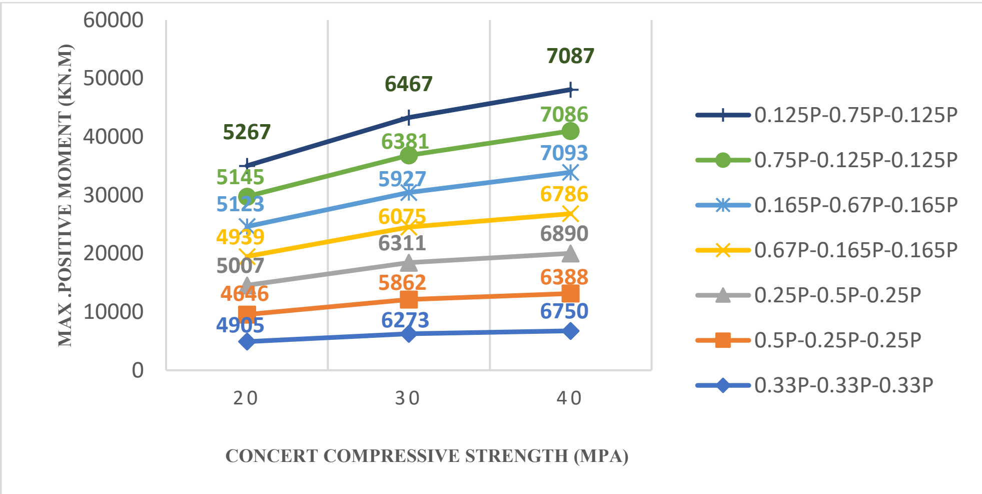


Figure 20. Maximum positive bending moment vs *f'c* for all groups

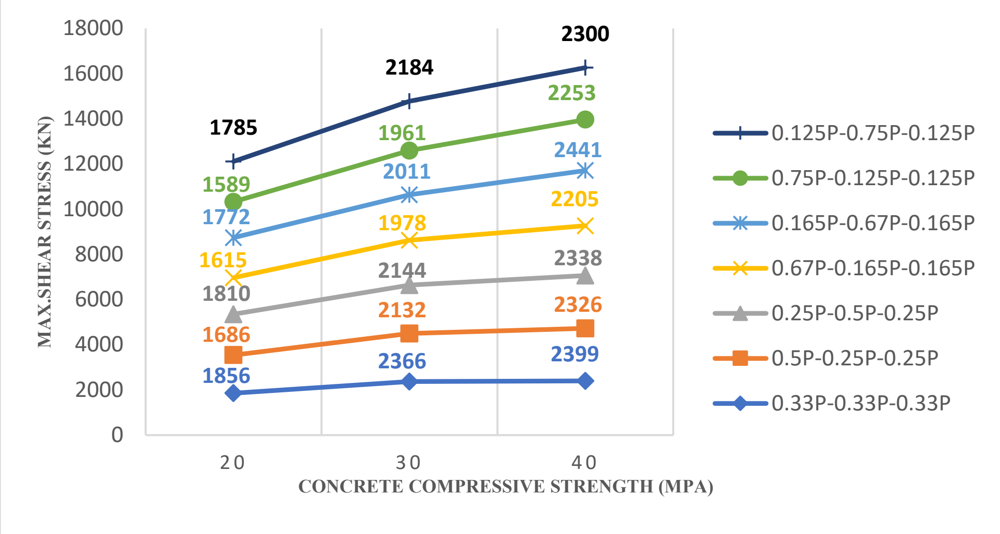


Figure 21. Maximum shear stress as a function of *f'c* for all groups

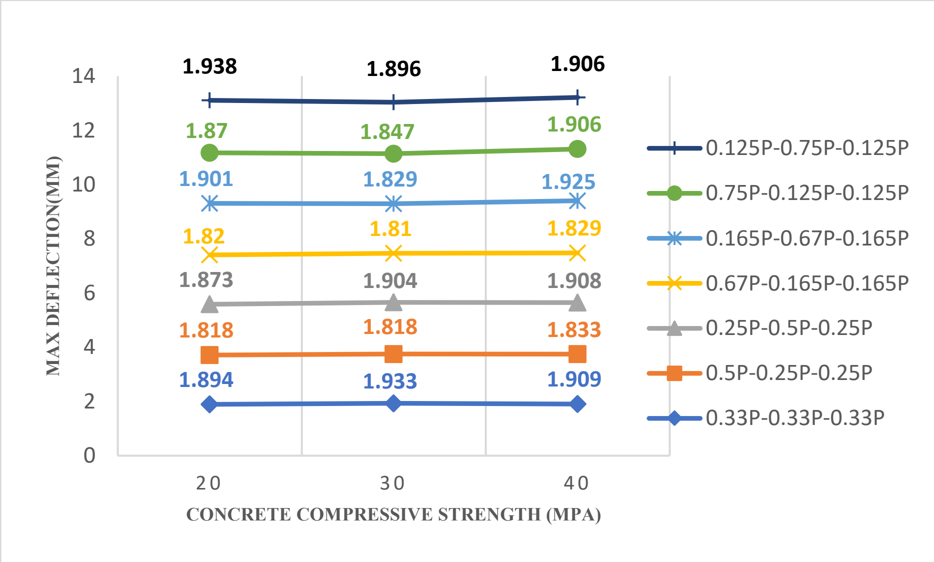


Figure 22. Maximum deflection as a function of *f'c* all groups

4. DISCUSSION OF RESULTS

Due to the formation of the load transfer truss (STM) in the deep beams between the loading and supporting points, the maximum load (0.75P, 0.67P, and 0.5P) being distant from the center affects the load capacity (2-11%). This occurs because the failure of the deep beam as a whole is a result of the failure of any one strut. More specifically, each strut will be exposed to one-third of the load if the load is present in the center, and the strut nearest the deep beam will be exposed to the maximum strength earlier than the matching strut farther away if the highest load is present at the edge of the beam. Because the strut's strength is almost constant in all load inequality scenarios, the load capacity won't vary significantly. Given that the bending moments in the shorter part of the span go up (0.7-5%), it makes sense that the midspan deflection of deep beams would decrease (0.2-6.4%) as the heaviest load goes away from the center of the span. Positive moments, on the other hand, go up (2-4.5%) as the load goes away from the center because they are the product of the length of the smaller span part and the highest reaction, which is closest to the load. Because shear stresses are concentrated on the side of the beam where the heaviest load is placed at the expense of the opposite side, they decrease (2.5-17%).

Finally, it must be mentioned that increasing the concrete compressive strength by about 50-100% increases the load capacity and reduces the deflection, maximum positive moment, and shear by about 29-37%, 0.7-2%, 27-38%, and 27-30%, respectively. The reason for this is due to the direct relationship between the concrete compressive strength of the strut and the compressive strength of the concrete, because the strut is, in the end, nothing but a compression member that resembles a column in much of its behavior.

5. CONCLUSION

The unequal load imposed on deep reinforced concrete beams may occur due to the architect's desire to do something unusual, or it may result from an error during implementation. This is not an easy matter because its consequences are complex. More specifically, the unequal load leads to asymmetry in the struts, which casts a shadow on the load capacity and deflection on the one hand, and the maximum positive moments and maximum shear stresses on the other hand. The analysis of twenty-one deep beams under different scenarios of load inequality led to many conclusions, the most important of which are:

1- The applied unequal load do not form symmetrical struts, so the strut with the highest load (the one to which the load is closest) fails. The failure of one of the struts leads to the failure of the deep beam directly, within lower bound theory. That is why, load inequality led to 2-11% decrease in load capacity.
2- When the load is unequal, the load distribution on the beam changes, causing stress to increase in some zones and decrease in others. This unbalanced distribution can reduce the maximum bending moment because the beam will fail with a load less than the central load. That caused a decrease in positive moments about 5.2-7%.
3- The shear values decrease as the load becomes unequal. It is true that the shear is higher on the side where the higher load is closer, but the applied failure load itself is small compared to the central failure load. Load inequality here led to 2.5-17% decrease in shear.
4- Because in deep beams, the total load capacity is often dependent on the strength of the strut, the change in the compressive strength of concrete directly affects the strength of the strut and thus the load capacity of the deep beam. Therefore, when the compressive strength of concrete increased from 20 to 40 MPa, the load capacity, deflection, positive moments, and shear increased by about 29-37%, 0.7-2%, 27-38%, and 27-30%, respectively.

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