

## **NON-LINEAR ANALYSIS OF CONTINUOUS COMPOSITE CONCRETE–STEEL BEAMS UNDER CYCLIC LOADINGS**

**Ali Laftah Abbas**

Engineering Collage, Diyala University

E-mail : alilaftast@yahoo.com

*(Received:18/1/2010 ; Accepted:22/2/2011)*

**ABSTRACT :-** Non-linear analysis of continuous composite concrete–steel beams under cyclic loadings has been investigated in this paper. A three-dimensional finite element analysis (FE) ANSYS computer program(ANSYS version 9.0) was conducted to investigate the nonlinear behaviour of this beam up to failure through the prediction of the values of slip, deflection along the spans of the composite beam for different number of loading cycles up to one million cycles of the load of range (52%of the ultimate load of the beam). The results obtained from analyzing a continuous composite concrete-steel beam under cyclic loadings show good agreement with available experimental results and other available analytical results. A parametric study is also conducted in this paper to study the influence of some parameters such as the number of loading cycles, and the amount of load ratio on the values of slip and deflection. This parametric study shows that as the number of loading cycles increased, slip and deflection values along the composite beam will increased due to reduction in strength of all components of the composite beam, and the absolute values of maximum slip and maximum deflection will reduce as the load ratio changes from negative to positive signs .

**Keywords:-** Composite beams, continuous beams, concrete-steel beam, deflection, cyclic loads, shear connectors, slip.

---

### **INTRODUCTION**

The use of steel and concrete composite structures account for the contribution of the two materials provided that a composite action exists between steel and concrete members. Reducing or preventing the relative displacement of concrete and steel section guarantees the

composite action. Shear connectors are used to provide this composite action. The efficiency of the shear connectors may be reduced due to repeated or cyclic stresses. This phenomenon of decreased resistance of materials to repeated stresses is called "fatigue", and the material test by the application of such stresses is called fatigue test. The first real attack of this problem was made by the German engineer Wohler in (1858). Since then a great deal of research has been conducted on fatigue of metals, and although this work has resulted in an ever increase understanding of the problem, there is yet no complete solution. It has been estimated from time to time that at least 75% of machine and structural failures have been caused by some form of fatigue. Experimental and theoretical works have been carried by several researchers in order to understand the behaviour of shear connectors under fatigue loads. Mainstone and Menzies (1967) performed static and fatigue tests on eleven push-out specimens using stud connectors, channel connectors, and bar connectors. Oehlers (1990) performed tests on the strength of stud connectors in composite bridge beams. Taplin and Grundy (1995) investigated the incremental slip behaviour of stud connectors. Johnson and Oehlers (1996) integrated static and fatigue design assessment of stud connectors in composite bridges, they used the results from Oehlers (1990) and presented a method for checking ultimate strength, taking account of fatigue damage. Oehlers et al. (1999) performed two static and four fatigue tests to investigate how stud shear connectors behave under cyclic load. Gattesco et al. (1997) suggested that inelastic behaviour of shear connectors change the structural response of a bridge in ways that cause reduction in load amplitude and load reversal.

Finally in order to validate the model and the solution of present study, a three-dimensional nonlinear finite element analysis was conducted to study the general behaviour of a continuous composite beam under cyclic loadings up to failure.

## **DERIVATION OF STRUCTURAL MATRICES**

For any general three-dimensional body, the element stiffness matrix for individual finite elements can be determined by using an energy principle, such as the principle of virtual work. This principle is concerned with the relationship that exists between a set of external loads and the corresponding internal forces that together satisfy the equilibrium and compatibility conditions or,

$$\delta W_{int} = \delta W_{ext} \dots\dots\dots (1)$$

where:

$W_{int}$  = strain energy (internal work).

$W_{ext}$  = external work (by applied loads).

$\delta$  = virtual operator.

The internal work after many derivations can be written as:

$$\delta W_{int} = \{\delta u\}^T \int_{vol} [B]^T [D][B] d(vol) \{u\} \dots\dots\dots (2)$$

where:

$[B]$  = strain- nodal displacement matrix, based on the element shape functions.

$\{u\}$  = nodal displacement vector.

The external work due to body forces and surface traction after many derivations is given by:

$$\delta W_{ext} = \{\delta u\}^T \left\{ \int_{vol} [N]^T \{b\} d(vol) - \int_S [N]^T \{t\} ds \right\} \dots\dots\dots (3)$$

where:

$\{b\}$  = the body forces (per unit volume).

$\{t\}$  = surface traction forces (per unit surface area).

$s$  = the part of the surface of the body where external traction is prescribed

Equation (1) can be written as;

$$\delta W_{int} - \delta W_{ext} = 0 \dots\dots\dots (4)$$

After substituting equations (2) and (3) into equation (4) the following is obtained:

$$\{\delta u\}^T \left\{ \int_{vol} [B]^T [D][B] d(vol) \{u\} - \int_{vol} [N]^T \{b\} d(vol) - \int_S [N]^T \{t\} ds \right\} = \{0\} \dots\dots\dots (5)$$

Since the relationship must be valid for any set of virtual displacements, and since  $\delta\{u\}^T$  is arbitrary or  $\delta\{u\}^T$  not equal to  $\{0\}$ , then equation (5) is written for the assemblage of elements as follows:

$$\{F\} = [K] \{u\} \dots\dots\dots(6)$$

where:

[K] Stiffness matrix of element assemblage (structure) and given by:

$$[K] = \sum_n \int_{vol} [B]^T [D][B] d(vol) \dots\dots\dots (7)$$

{F} is the assemblage of external nodal forces vector and given by:

$$\{F\} = \sum_n \int_{vol} [N]^T \{b\} d(vol) + \sum_n \int_S [N]^T \{t\} ds \dots\dots\dots (8)$$

and n is total number of elements.

The differential volume d (Vol) for three-dimensional body in x, y and z-coordinates can be written as:

$$d (vol) = dx dy dz \dots\dots\dots(9)$$

The above equation can be transformed into local coordinates  $\xi, \eta$  and  $\zeta$  as:

$$d (vol) = |J|. d\xi. d\eta. d\zeta \dots\dots\dots (10)$$

where |J| is the determinant of the Jacobian matrix

The stiffness matrix for an element in x, y and z coordinates is given by

$$[k]_e = \int_{vol} [B]^T [D][B] d(vol)_e \dots\dots\dots(11)$$

The limits of the integration in the local coordinates  $\xi, \eta$  and  $\zeta$  become -1 and +1, and then element stiffness matrix becomes:

$$[k]_e = \int_{-1}^{+1} \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [D][B] |J| d\xi d\eta d\zeta \dots\dots\dots(12)$$

The above equation can represent the element stiffness matrix for (solid 65) element which is used in modelling the concrete slab of the composite beam.

The element stiffness matrix for (shell 43) which is used in modelling the steel I-beam can be written as:

$$[k]_e = \int_{-1}^{+1} \int_{-1}^{+1} [B]^T [B][D] |J| d\xi d\eta \dots\dots\dots (13)$$

In order to resist slip at steel-concrete interface, nonlinear spring element (combin 39) is used for this purpose. The element stiffness matrix for this element is given by:

$$[K^e] = K^{tg} \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots (14)$$

where:

$K^{tg}$  is the slope of active segment of the force-deflection curve of this element.

The element stiffness matrix for linear spring (combin 40) which is used to prevent the vertical uplift as given as:

$$[K_e] = K \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dots\dots\dots(15)$$

Where

$$K = \begin{cases} K_1 + K_2 & \text{if slider was not sliding} \\ K_2 & \text{if slider was sliding} \end{cases}$$

## MATERIALS PROPERTIES

In this study, concrete element is modeled by using 3-dimensional structural solid element 8-node brick element (solid 65). Each of the eight nodes has three degrees of freedom, translations in the nodal x, y and z-direction. The geometry and node locations for this element are shown in Figure (2).

This element which is offered by ANSYS computer program can simulate the nonlinear property of concrete. This element behaves as a linear elastic material until the stress reaches the tensile or compressive strength at an integration point, then the stress-strain relationship of the element will be modified by introducing a plane of weakness in the direction normal to the stress to represent the cracking.

The failure surface of concrete can be specified with a minimum of two constants  $f_c$  and  $f_t$  or specified with all five parameters of concrete strength by Willam and Warnke criterion in addition to an ambient hydrostatic stress state.  $f_c$  and  $f_t$  can be determined from simple tests. The other three parameters default to Willam and Warnke. After cracking, the tensile stress of the concrete element is set to zero in the direction normal to the crack plane but the concrete between cracks can take tensile stresses due to bond with steel bars. Typical shear transfer coefficients range from zero to one, with zero value representing a smooth crack (complete loss of shear transfer) and one representing a rough crack (no loss of shear transfer). This specification may be used for both the closed and open crack. The higher values of the shear transfer coefficient were used to avoid convergence problems which occur when the shear transfer coefficient  $\beta$  in  $\beta G$  for open crack drops below 0.2. No serious deviation of the response occurs with the change of the coefficient; therefore, the coefficient for open crack was set to 0.3. The materials properties of the steel beam and reinforcement were specified with a typical bilinear idealization in both tension and compression.

The steel beam can be modeled by using a three-dimensional 4-node quadrilateral shell element (shell 43). This element is defined by four nodes. Each node has five degrees of freedom: translation in the nodal x, y and z direction and rotations in the nodal x and y-direction. The drilling rotation in z-direction is omitted. Atypical shell element is considered. The local coordinates of the shell element is supposed to lie in the mid-surface of the element, which is therefore the  $\zeta = 0$  surface. The three local coordinates  $\xi$ ,  $\eta$ , and  $\zeta$  are also assumed to have range varying between -1 and +1 on the respective faces of the element

(Zienkiewicz 1971). The external face of the element is curved while straight lines generate the section across the thickness. Pairs of points  $i_{top}$  and  $i_{bottom}$ , each with Cartesian coordinates, prescribe the shape of the element. Figure (3).

The shear studs which restrain the longitudinal slip can be modelled by nonlinear spring elements of zero length (combin 39) to resist slip. This element is defined by two nodes and by a force-deflection curve in which force versus relative translation is specified. The geometry, node locations, and the nonlinear force-deflection for this element are shown in Figure (4). This element has nonlinear generalized force-deflection capability that can be used in any analysis, and has longitudinal or torsional capability in one, two or three dimensional application. It also has large displacement capability for which there can be two or three degrees of freedom at each node (translation in the nodal x, y, z-directions).

While shear connectors which restrain the vertical uplift can be modelled by using combination element (combin 40). The geometry of this element is shown in Figure (5).

This element is a combination of a spring-slider and damper in parallel, coupled to gap in series. A mass can be associated with one or both nodal points. The element has one degree of freedom at each node, either a nodal translation, rotation, pressure, or temperature. The mass, spring, slider, damper, and/ or the gap may be removed from the element which is defined by two nodes.

The force-deflection relation for this element under initial loading is as shown in Figure (6) (for no damping).

where:

$$F_1 = \text{force in spring 1}$$

$$F_2 = \text{force in spring 2}$$

$$K_1 = \text{stiffness of spring 1}$$

$$K_2 = \text{stiffness of spring 2}$$

$$u_{gap} = \text{initial gap size}$$

$$u_I = \text{displacement at node I}$$

$$u_J = \text{displacement at node J}$$

$$F_s = \text{force in spring 1 to cause sliding}$$

## BEAM IN STUDY

Teraskiewicz (1967) tested a two-span continuous composite beam of 6.7m long and under two-point loading. The beam geometry, cross-section dimensions and load setup are shown in Figure (7). Shear connection is provided with equally spaced shear studs of 9 mm diameter and 50 mm height and with two studs in a row. The material properties of the composite beam are shown in Table (1). The shear force-slip relationship proposed by Yam and Chapman (Yam and Chapman 1971) is used:

$$Q = 32 (1 - e^{-4.75 \text{ ucs}}) \dots\dots\dots (16)$$

where Q is the shear force in kN and ucs is the slip in mm

To examine the effect of cyclic loads on a continuous composite beam, Teraskiewicz continuous composite beam is taken to be subjected to a load at each mid-span. The range of this load is 52% of the ultimate load of the beam (72.8 kN).

## ANALYSIS OF RESULTS

In order to analyze the behaviour of continuous composite concrete-steel beams under cyclic loadings, ANSYS computer program is used for this purpose by taking into consideration the degradation of the properties of the materials of the composite beam under cyclic loadings.

A beam experimentally tested under static load is considered. The test results are compared with those obtained by Al-Aquly (2002). Furthermore the effects of some parameters on the behaviour of this beam such as the number of loading cycles, the amount of load ratio on the values of slip and deflection are also studied.

The three-dimensional finite element representation mesh for the whole beam (ignore symmetry) using the (ANSYS 9.0) software is shown in Figure (8). Concrete slab is idealized by using 3312 eight-node brick elements (solid 65- reinforced concrete element) and the steel beam by 1012 four-noded shell elements (shell 43-plastic shell). Shear connectors are idealized by 92 two-noded nonlinear spring (combin 39) to resist slip and 92 two-noded linear spring (combin 40) to resist uplift separation. The total number of nodes in the above idealization is 5952 node .

Figure (9) shows the deflection shape of Teraskiewicz composite beam at load P=122 kN (87% of the ultimate load of the beam) at mid-spans by using the experimental results

obtained by Teraskiewicz (1967) and other three methods of representation which are the present study by using ANSYS computer program (version 9.0), the composite beam element and the three-dimensional analysis, obtained by Al-Aquly (Al-Aquly 2002), by using the program (QHA2) and the software (ANSYS 5.4), respectively.

The deformed and undeformed shapes of this beam are shown in Figure (10).

Figure (11) shows the slip distribution along Teraskiewicz composite beam at load ( $P=122$  kN) at mid-spans. The curves show that the maximum slip is located about 1000 mm from the mid support.

An ultimate load of 151 kN was obtained during the actual test, at that stage the shear connector failure was observed in a region between the central support and the left hand mid-span. The present study predicted an ultimate load of 140 kN with the predicted mode of failure involving failure of shear connectors within the same region as observed in the experiments.

Figures (12) and (13) show the variation of slip and deflection along the beam axis at load  $P=72.8$  kN with the increased number of loading cycles which varied from one thousand to one million cycles after which failure occurred. The curves show that the maximum slip is located about 1000 mm from the middle support and the deflection values varied from zero at support to maximum value at mid-spans.

The general behaviour encountered is an increase in maximum slip and deflection values as the number of loading cycles increase because of reduction in the materials strength due to fatigue.

Figures (14) and (15) show the variation of maximum slip and maximum deflection with the number of loading cycles for different load ratios ( $R$ ). This ratio can be defined as the ratio between the minimum and maximum values of the load that has been applied to the structural element. Through these two figures, the load ratio have been varied from  $-0.75$  to  $+0.75$  to cover wide range of loading application possibilities. It can be seen that the absolute values of maximum slip and maximum deflection will reduce as the load ratio changes from negative to positive signs. The positive sign of loading ratios means that the loading will be in the same direction but with variable values while negative sign of loading ratio means a reversal of loading will occur. Hence, the resulting slip and deflection values will be reduced in the positive sign region of stress ratio.

## CONCLUSIONS

Based on the results of the available experimental test and other available analytical results with the comparison with the results obtained from the finite element analysis by ANSYS(version 9.0) it is shown that the computer modeling by used efficiently to predict the behaviour of such composite beam.

The results obtained from finite element analysis shows good accuracy in comparison with available experimental results under static load.

With a partial interaction, the effect of cyclic loading will be magnified due to the presence of slip at the interface. Shear stud connectors will undergo reduction due to effect of cyclic loading. For load of 52% of the ultimate load ( $P=72.8$  kN), the maximum slip increased from 0.31 to 0.53mm when the cycles are increased from  $10^3$  to  $10^6$  respectively. A low number of loading cycles less than about one thousand cycles has little effect upon the overall behaviour of the composite beam, while loading cycles over one thousand cycles will reduce the stud strength due to fatigue effect. Accordingly, the values of slip and deflection are increased as the number loading cycles is increased and the absolute values of maximum slip and maximum deflection will reduce as the load ratio changes from negative to positive signs for constant number of loading cycles.

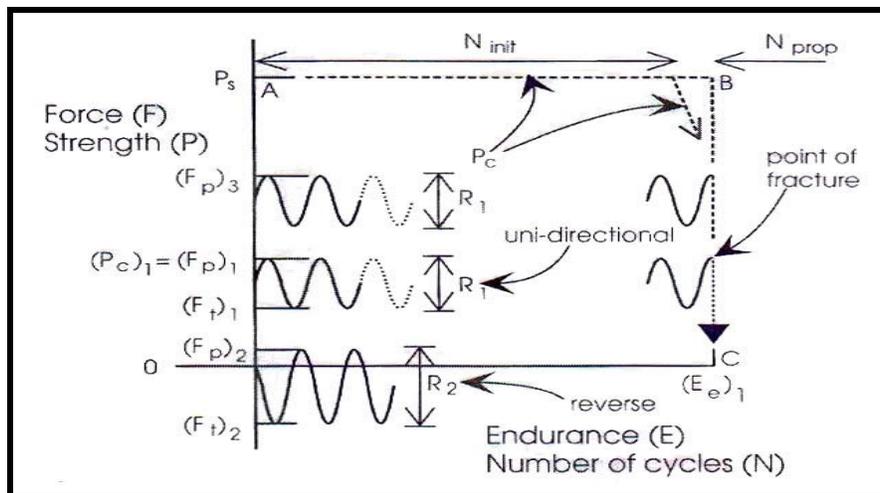
## REFERENCES

1. "ANSYS Manual Version 9", SAS IP, U.S.A., 2004.
2. Al-Aquly, Q. A., "Analysis of Continuous Composite Concrete-Steel Girders with Partial Interaction" M.Sc. thesis, Department of Civil Engineering, College of Engineering, University of Baghdad, December (2002),pp.140.
3. Gatesco, N., Giuriani, E., and Gubana, A., "Low-Cycle Fatigue Ttest on Stud Shear Connectors" Journal of Structural Engineering, ASCE, 123 (2), 1997, pp. 145-150.
4. Johnson, R. P. and Oehlers, D. J., "Integrated Static and Fatigue Design or Assessment of Stud Shear Connections in Composite Bridges", The Structural Engineer. 74 (14), (1996) p 236-240.
5. Kayir, H. "Methods to Develop Composite Action in Non-composite Bridge Floor Systems: Fatigue Behaviour of Post-Installed Shear Connectors", MS Thesis. Department of Civil, Architectural and Environmental Engineering, University of Texas at Austin, (2006).

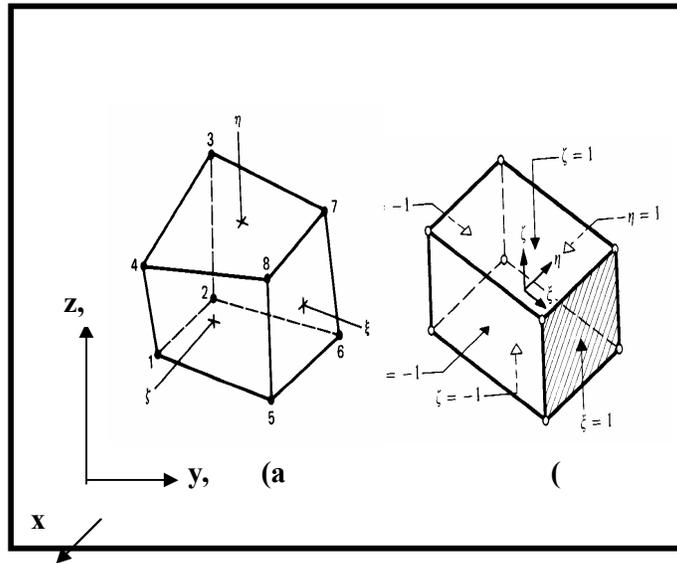
6. Mainstone, R. J. and Menzies, J. B., "Shear Connectors in Steel-Concrete Composite Beams for Bridges", *Concrete*, 1967, 1, Sept. No. 9, pp. (291-302).
7. Oehlers, D. J., "Deterioration in Strength of Stud Connectors in Composite Bridge Beams", *Journal of Structural Engineering*. 116 (12), (1990) p 3417-3431.
8. Oehlers, D. J., Seracino and Yeo "Reverse -Cyclic Load Tests on Stud Shear Connectors", (1999)
9. Taplin, G. and Grundy, P., "The Incremental Slip Behaviour of Stud Connectors", *Proceedings of the Fourteenth Australasian Conference of the Mechanics of Structures and Materials*, Hobart, (1995).
10. Teraskiewicz, J. S., "Static and Fatigue Behaviour of a Simply Supported End Continuous Beams of Steel and Concrete" Ph.D. thesis, London University, 1967.
11. Willam, K., and Warnke, E., "Constitutive Model for Triaxial Behaviour of Concrete", *Proceedings, International Association for Bridge and Structural Engineering*, Vol. (19), ISMES, pp. 174, Bergamo, Italy, 1975, (cited by Ref.2).
12. Yam, L. C. P. and Chapman, J. C., "The Inelastic Behaviour of Simply Supported Composite Beams of Steel and Concrete", *Proceedings, Institution of Civil Engineers*, U.K., Part 2, Vol. 41, December 1965, pp. 651-683.
13. Yam, L. C. P. and Chapman, J. C., "The Inelastic Behaviour of Continuous Composite Beams of Steel and Concrete", *Proceedings, Institution of Civil Engineers*, U.K., Vol. 53, 1972, pp. 487-501.
14. Zienkiewicz, O. C., "The Finite Element Method in Engineering Science", McGraw-Hill, London 1971.

**Table (1):** Material Properties of Composite Beam Tested by Teraskiewicz (1967)<sup>(10)</sup>.

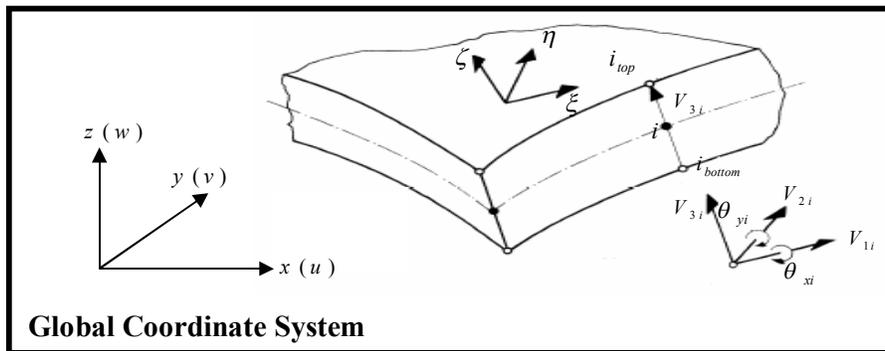
Material	Properties
Steel beam	Yield stress = 285 MPa (tested)
	Young's modulus = 200000 MPa (assumed)
	Strain hardening modulus = 1000 MPa (tested)
	Possion's ratio $\nu = 0.3$
Concrete	$f_c = 48\text{MPa}$
	Young's modulus = 27600 MPa (assumed)
	Possion's ratio $\nu = 0.15$
	Cracking stress $f_{cr} = 4.8\text{ MPa}$ (tested)
Reinforcement	Yield stress = 310 MPa (tested)
	Young's modulus = 200000 MPa (assumed)
	Possion's ratio $\nu = 0.3$
	Top transverse bars = $\phi 8 @ 102\text{ mm c/c}$
	Top longitudinal bars = $\phi 8 @ 65\text{ mm c/c}$
	Bottom transverse bars = $\phi 4.8 @ 204\text{ mm c/c}$
Shear stud connector	Spacing = 146mm
	Diameter $\times$ height = $9 \times 50\text{ mm}$
	Number of rows = 2



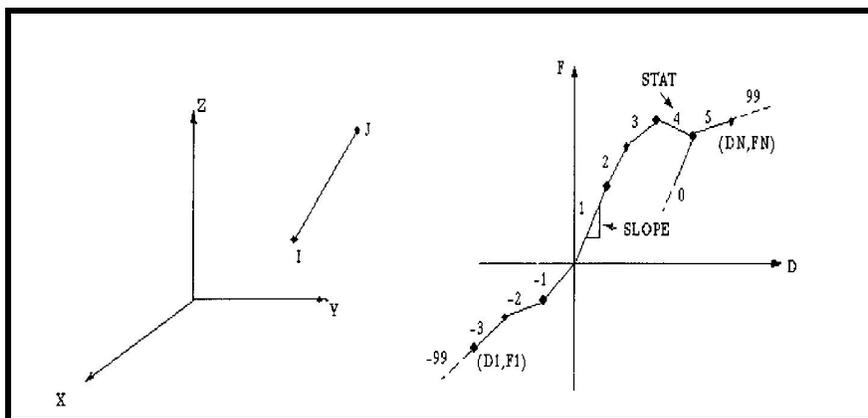
**Fig.(1):** Schematic figure of fatigue strength during fatigue testing<sup>(5)</sup>.



**Fig. (2):** The eight-node brick element (solid 65) by ANSYS<sup>(1)</sup>.  
 (a) Actual element in global coordinate system    (b) In local coordinate system



**Fig.(3):** Shell element.



**Fig. (4):** Nonlinear spring element<sup>(1)</sup>.

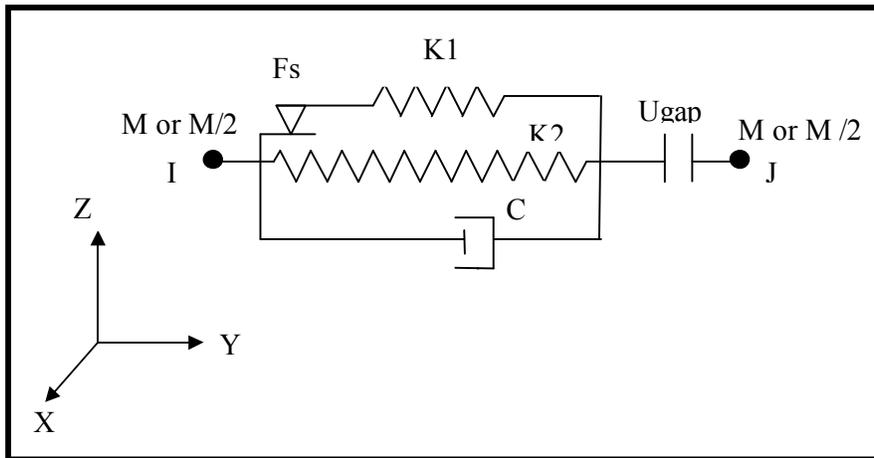


Fig.(5): Linear spring element<sup>(1)</sup>.

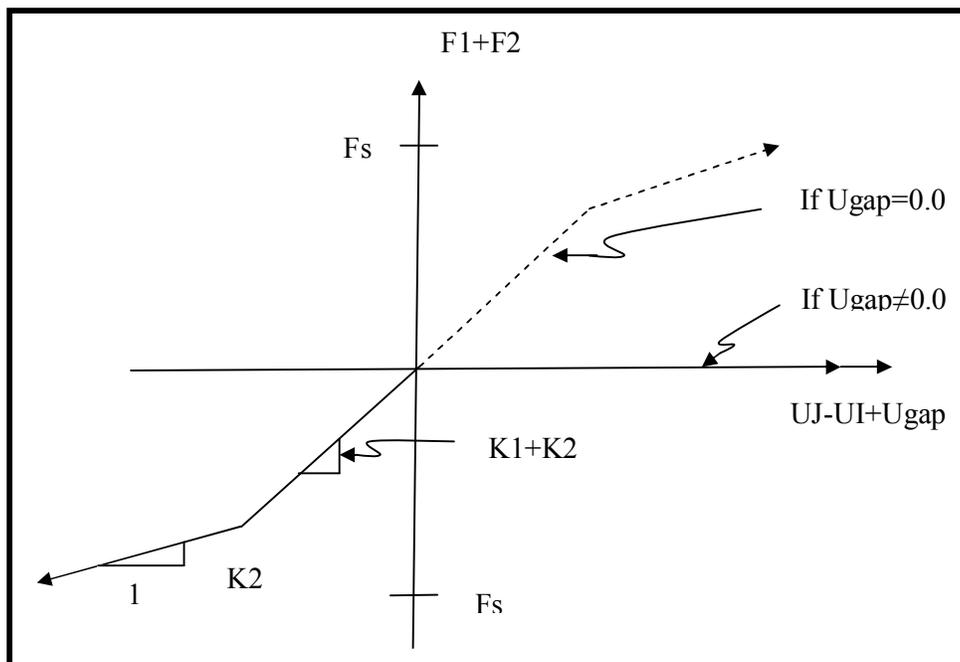


Fig.(6): Force -deflection relationship<sup>(1)</sup>.

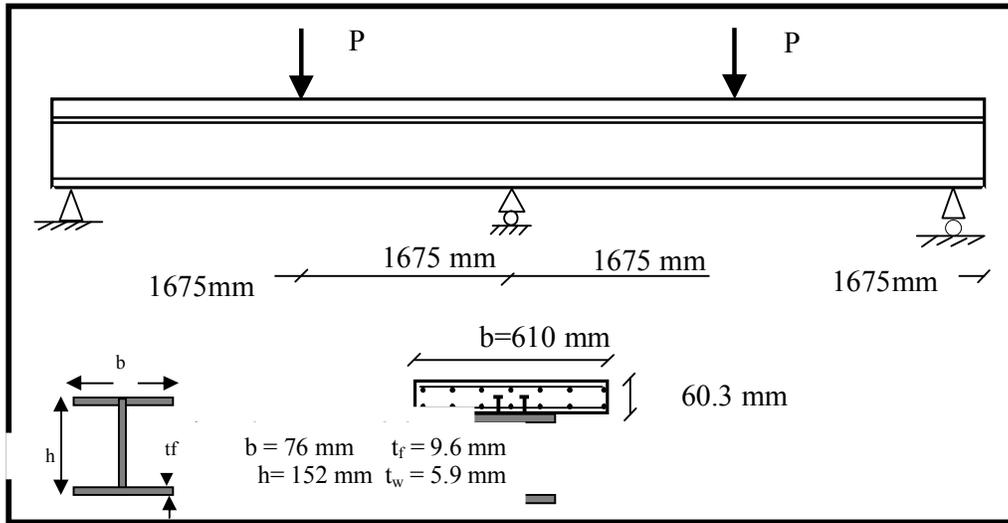


Fig.(7): continuous composite beam tested by Teraskiewicz (1967)<sup>(10)</sup>.

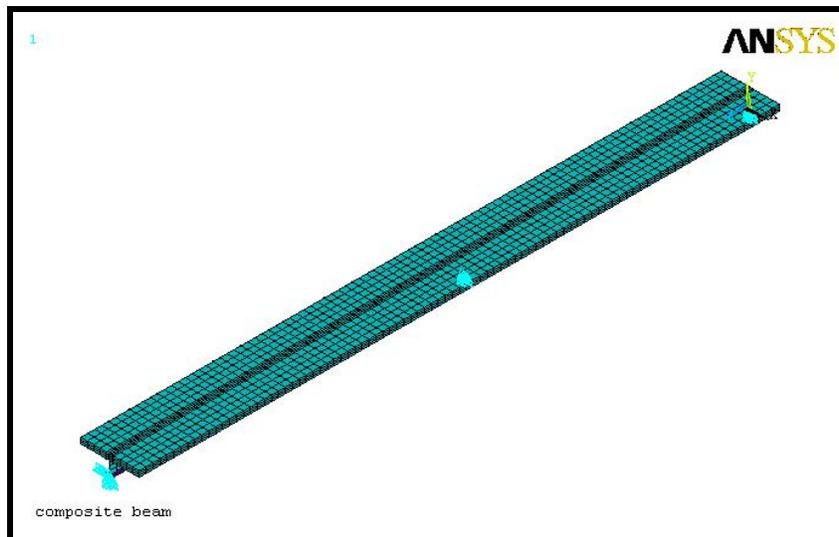


Fig. (8): Composite beam with boundary conditions.

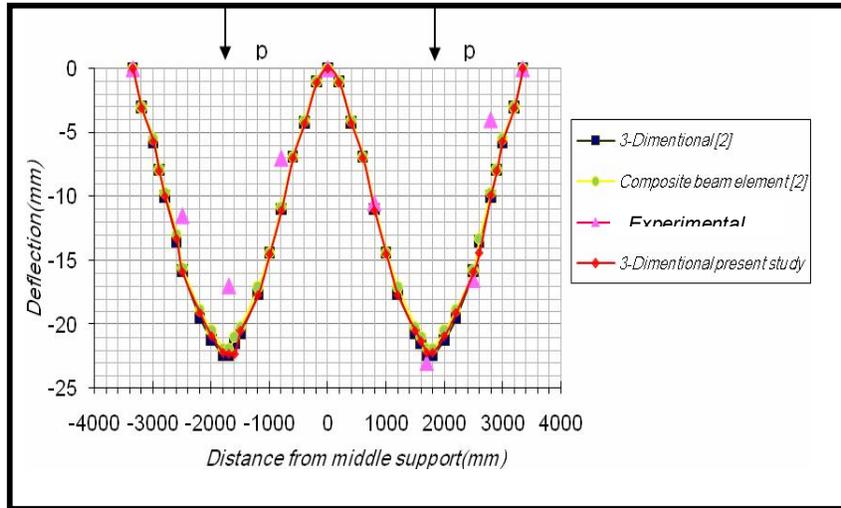


Fig. (9): Deflected shape of Teraskiewicz composite beam at load  $P = 122\text{kN}$  at mid- spans.

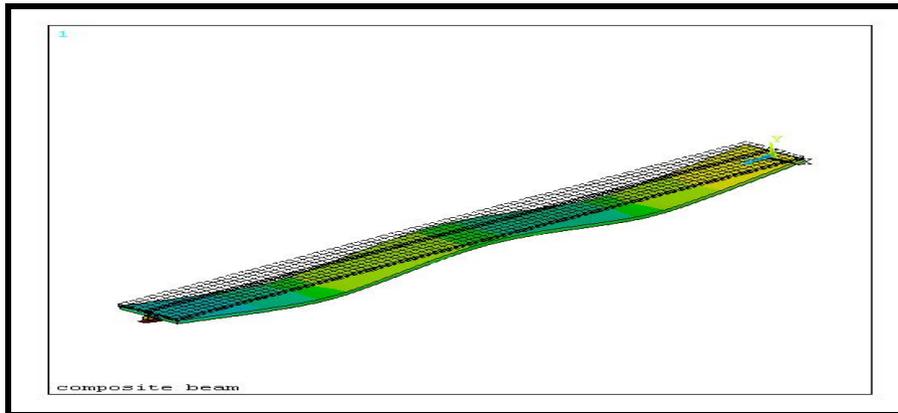


Fig.( 10 ): Deformed and undeformed shapes of composite beam by ANSYS.

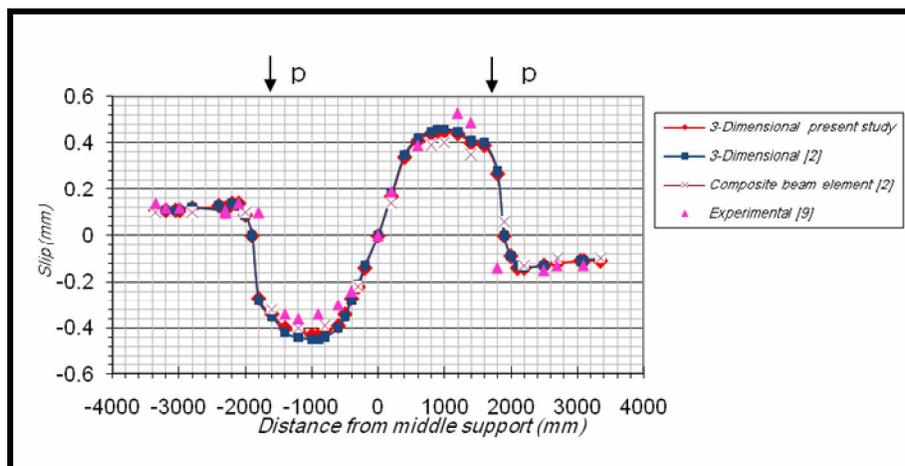


Fig.(11): Slip distribution along Teraskiewicz composite beam at load  $P = 122\text{kN}$  at mid- spans.

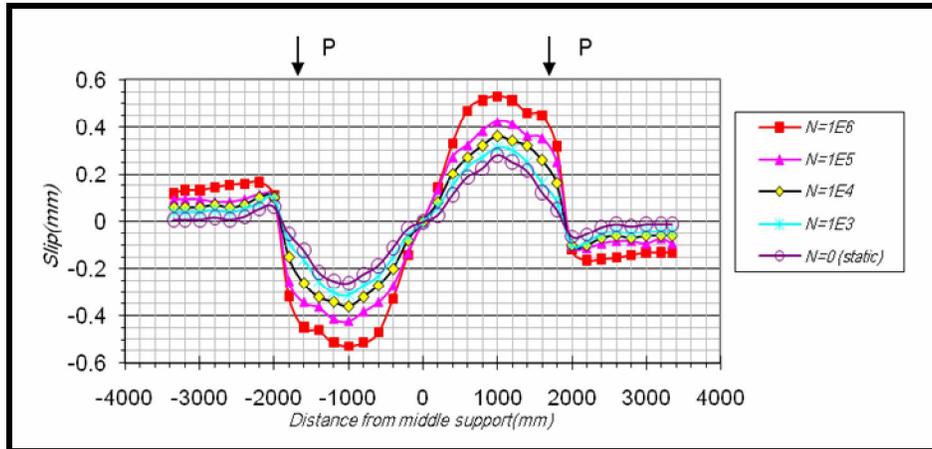


Fig.(12): Variation of slip along beam axis at load  $P = 72.8\text{kN}$  with number of loading cycles.

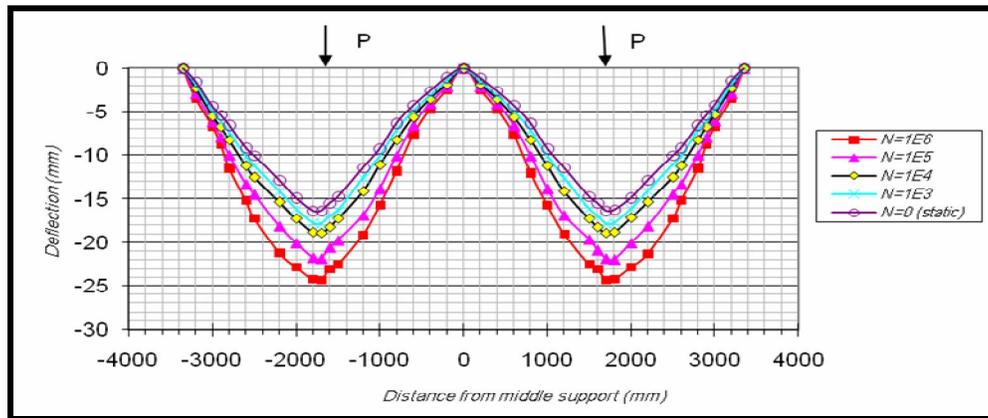


Fig. (13): Variation of deflection along beam axis at load  $P = 72.8\text{kN}$  with number of loading cycles.

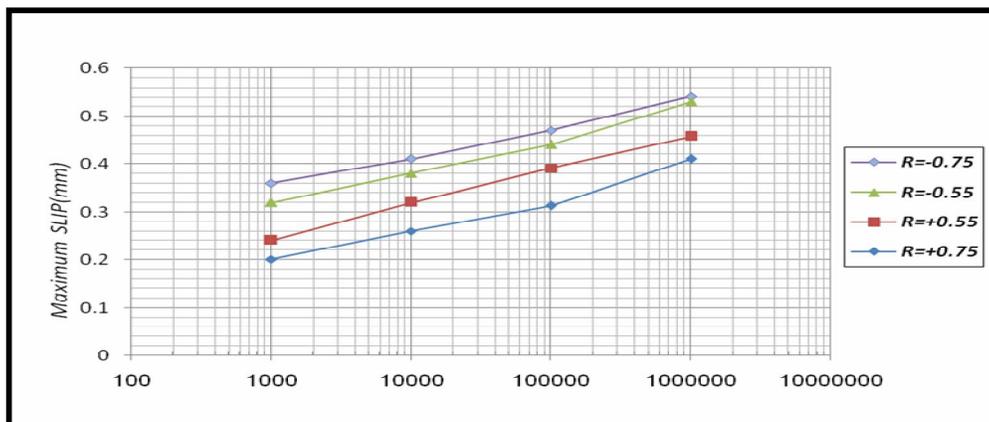


Fig. (14): Variation of maximum slip with the number of loading cycles for different load ratios.

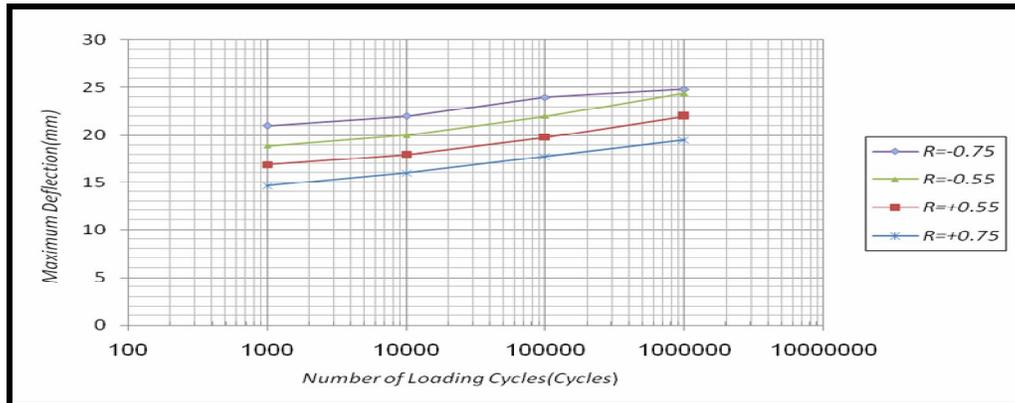


Fig. (15): Variation of maximum deflection with the number of loading cycles for different load ratios .

## السلوك اللاخطي للعتبات المستمرة المركبة من كونكريت-حديد تحت تأثير الأحمال الدورية

د . علي لفته عباس

مدرس

كلية الهندسة\_ جامعة ديالى

### الخلاصة

ان السلوك اللاخطي للعتبات المركبة المستمرة المكونة من الكونكريت والحديد تحت تأثير أحمال متكررة تم دراستها في هذا البحث و قد تم استخدام البرنامج (ANSYS version 9.0) لهذا الغرض. لقد تم استخدام طرق التحليل بواسطة العناصر المحددة لمعرفة السلوك اللاخطي لهذه العتبة إلى حد الفشل من خلال إيجاد قيم الانزلاق والود على طول العتبة ولإعداد مختلفة من دورات التحميل إلى حد مليون مرة تكرار للحمل المسلط والذي نسبته (52%) من الحمل الأقصى لهذه العتبة). لقد تمت مقارنة النتائج المستحصلة من التحليل بواسطة العناصر المحددة مع النتائج العملية المتوفرة وكذلك مع نتائج التحليل الأخرى المتوفرة، وقد أظهرت نتائج المقارنة حصول توافق جيد. وأخيرا تمت دراسة تأثير بعض المتغيرات مثل عدد دورات التحميل و نسبة الحمل المسلط على قيم الانزلاق والود. أن تأثير بعض المتغيرات أظهرت بأن قيم الانزلاق والود سوف تزداد مع زيادة عدد دورات التحميل المتكررة بسبب النقصان الحاصل في مقاومة كل العناصر الداخلة في تكوين العتبة المركبة وان القيم القصوى للانزلاق والود سوف تقل عندما تتغير نسبة الحمل المسلط من القيم السالبة إلى القيم الموجبة.

الكلمات الدالة: العتبات المركبة، العتبات المستمرة، العتبة المكونة من الكونكريت والحديد، الأود، الأحمال المتكررة، روابط القص، الانزلاق.